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OF

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LATE SADLERIAN PROFESSOR OF PURE MATHEMATICS IN THE UNIVERSITY OF CAMBRIDGE.

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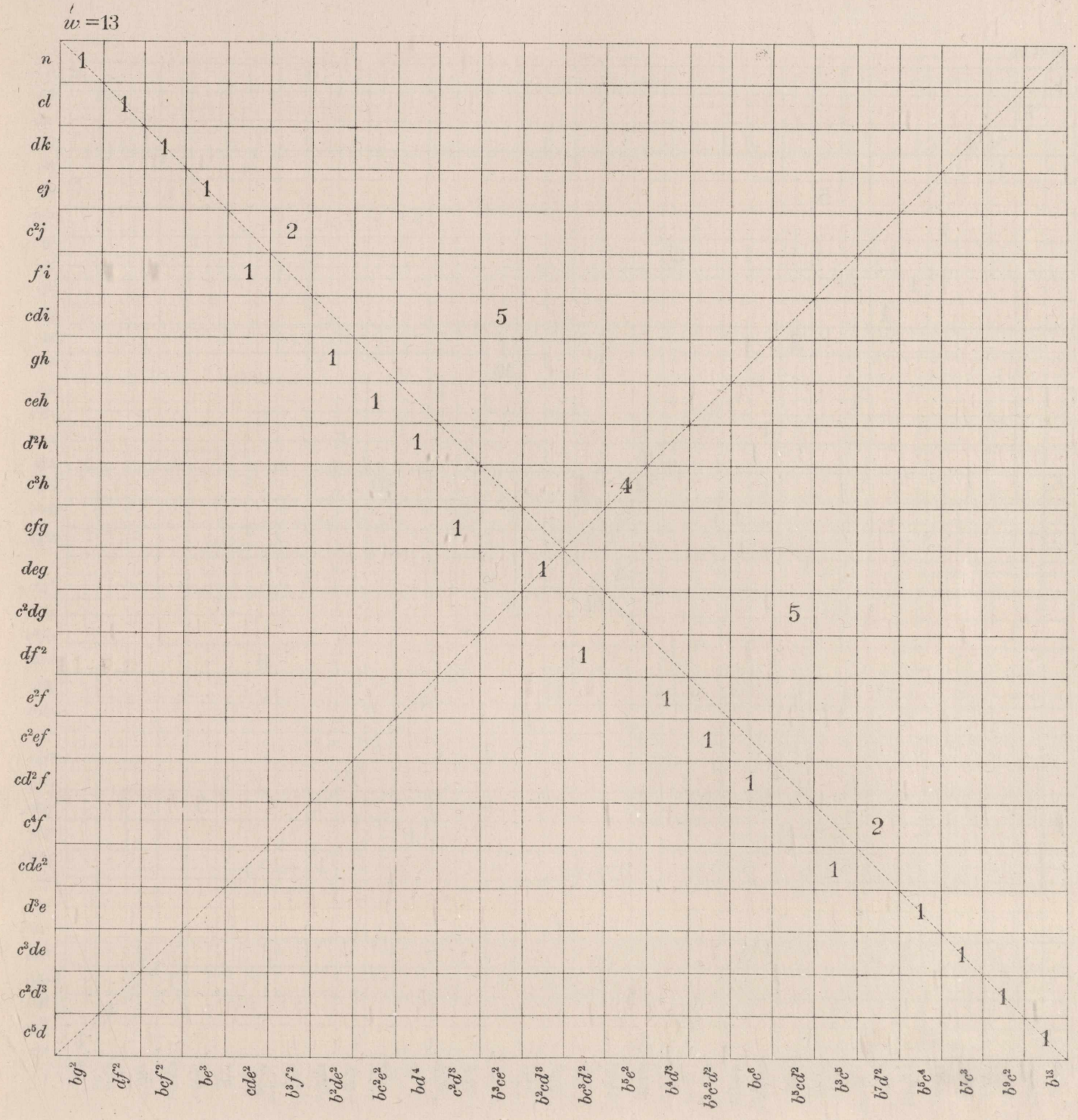

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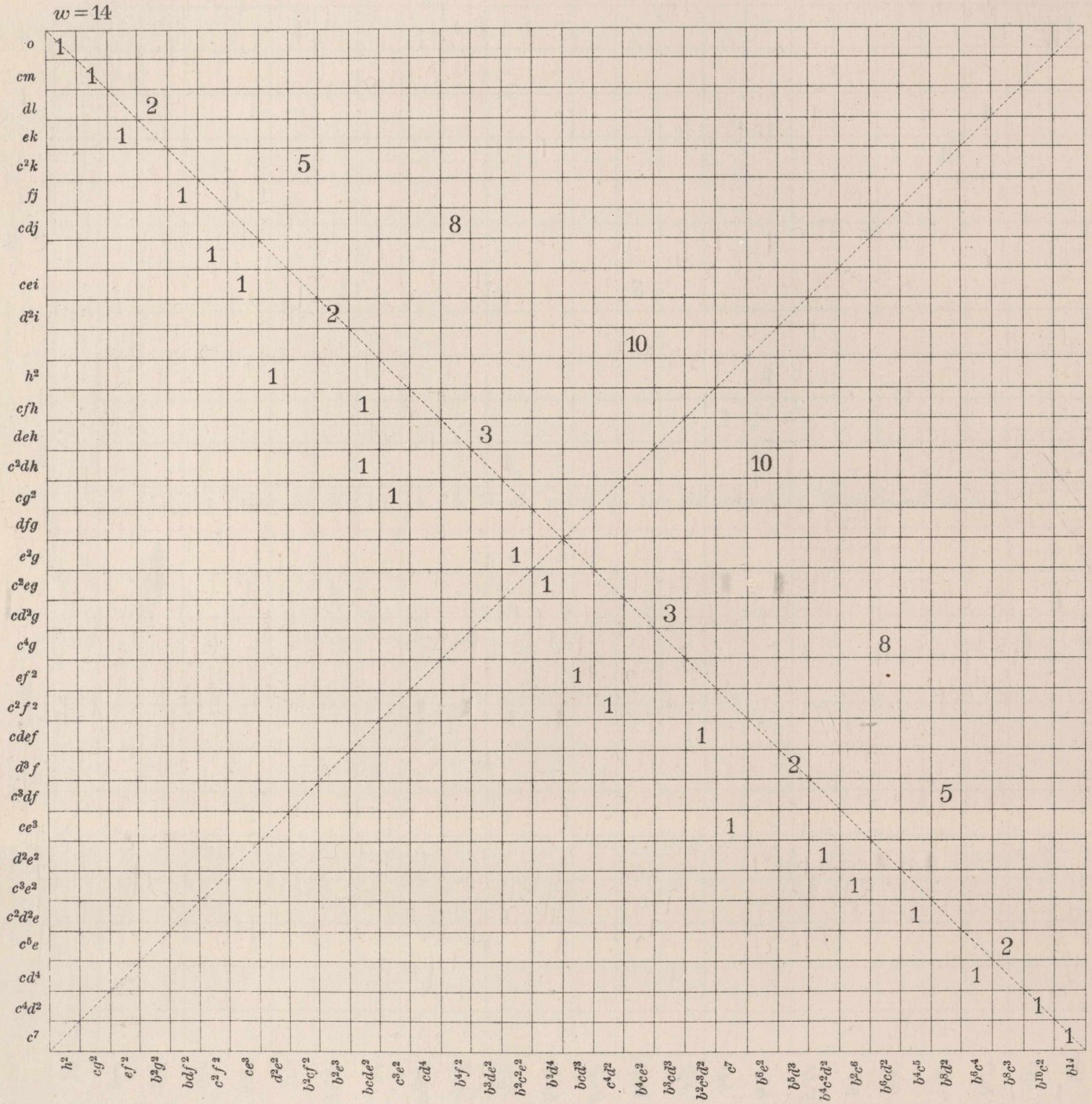

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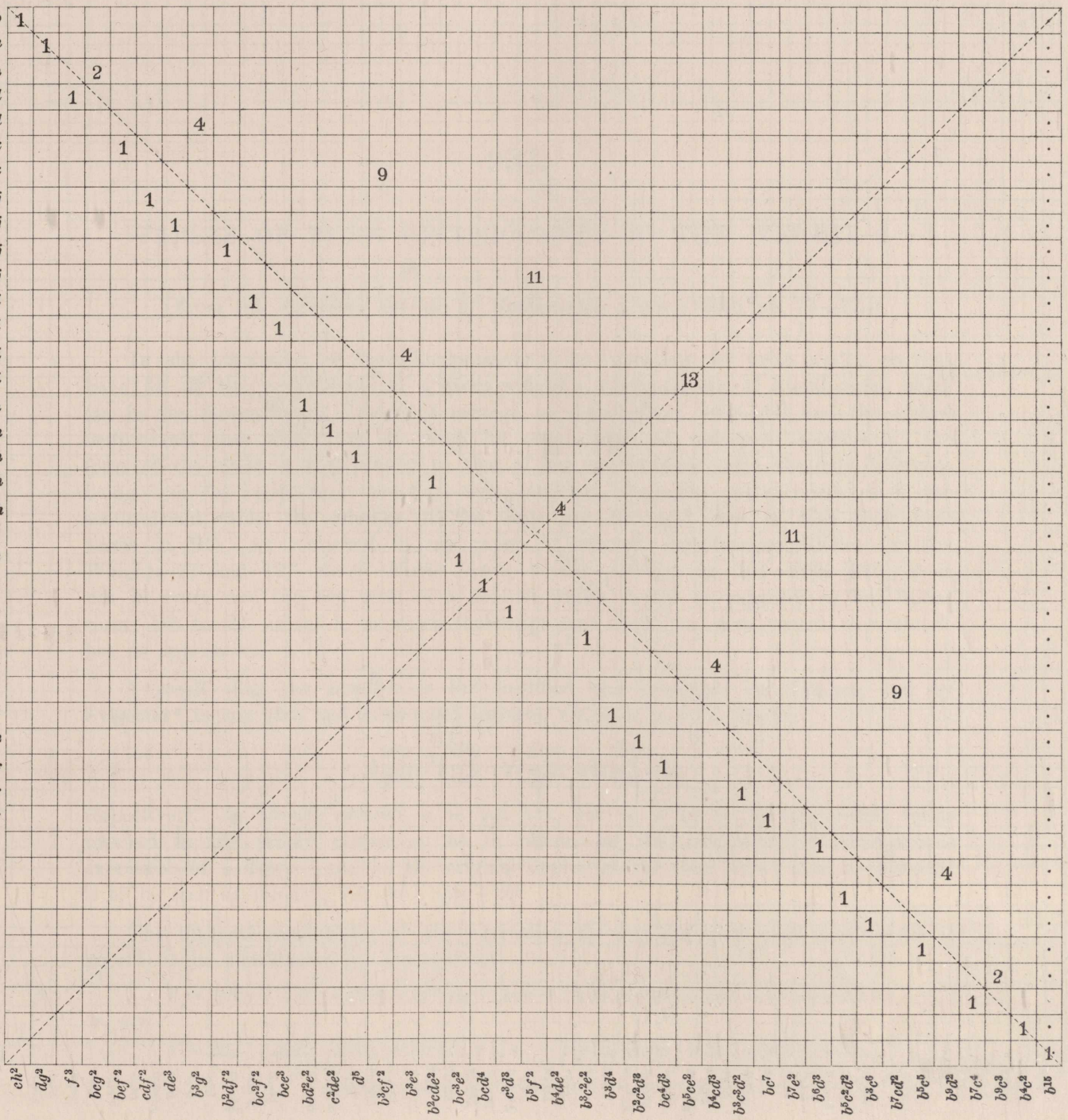

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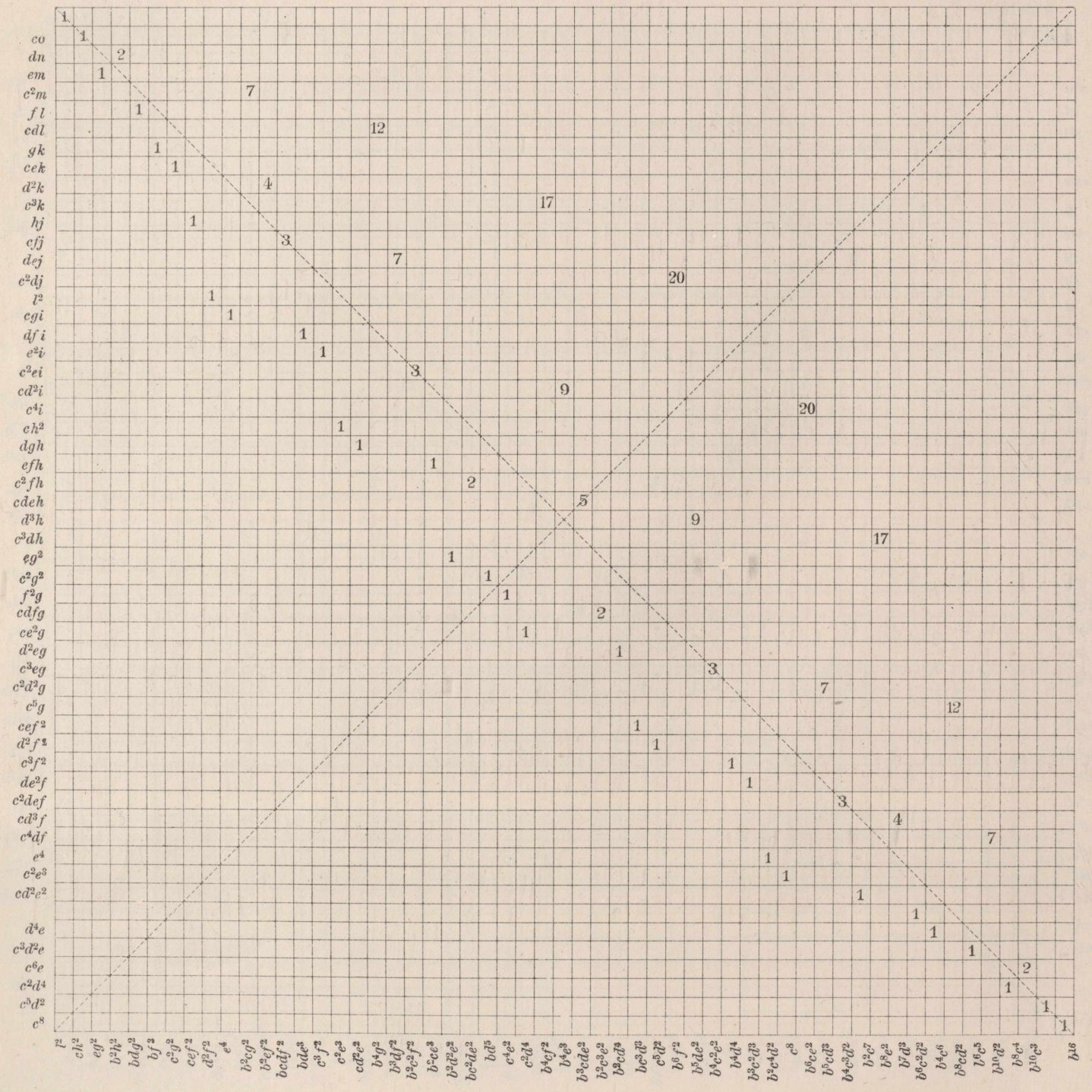

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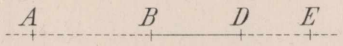

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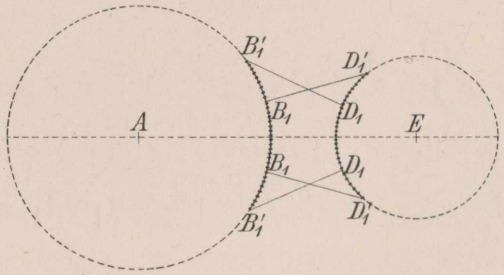

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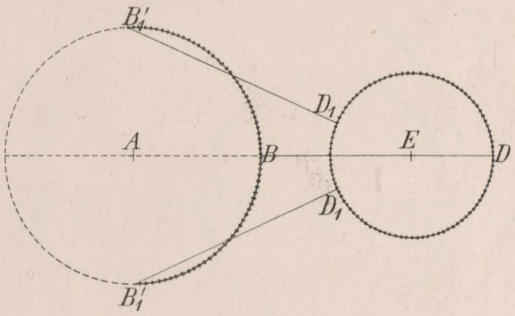

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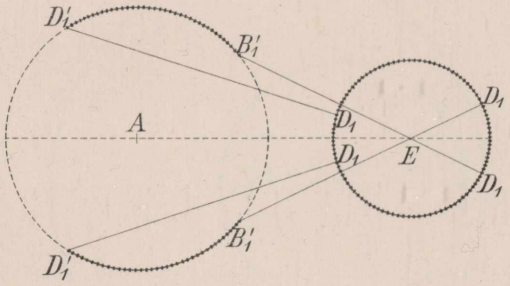

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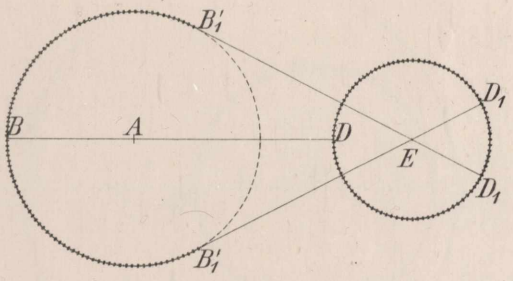

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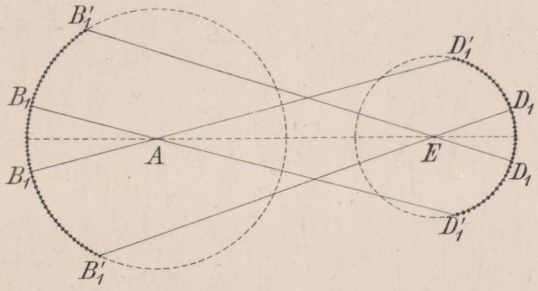

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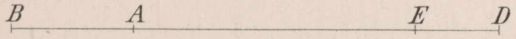

Fig. 4. $c=a+b+d$

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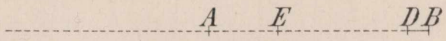

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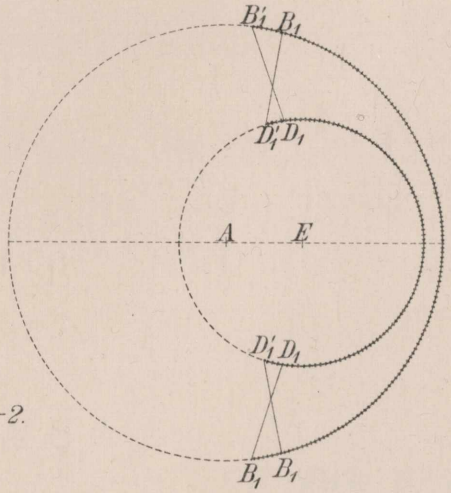

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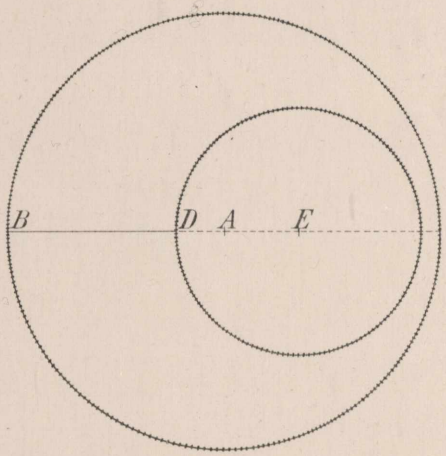

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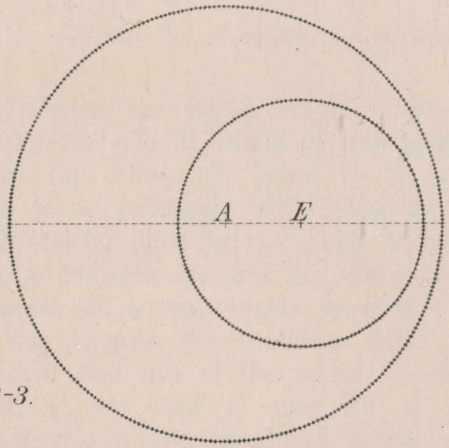

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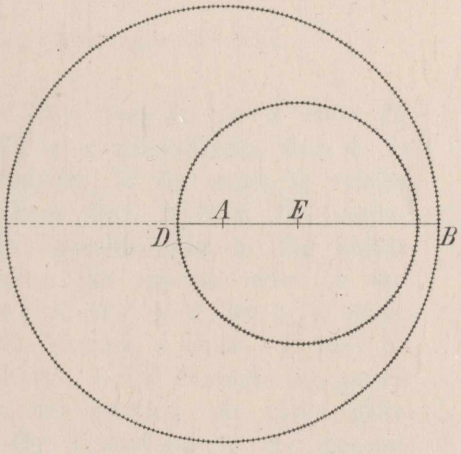

Fig. 3.

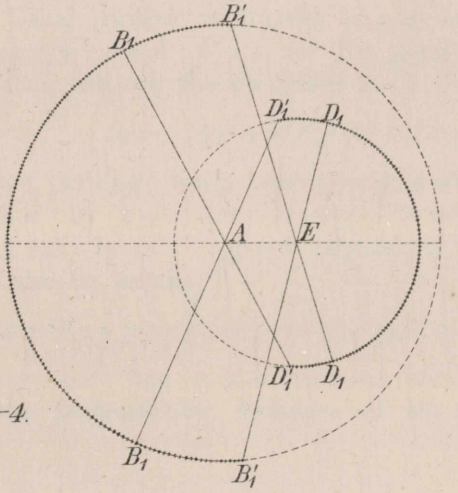

Fig. 3-4.

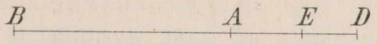

Fig. 4.

Interior Series.

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