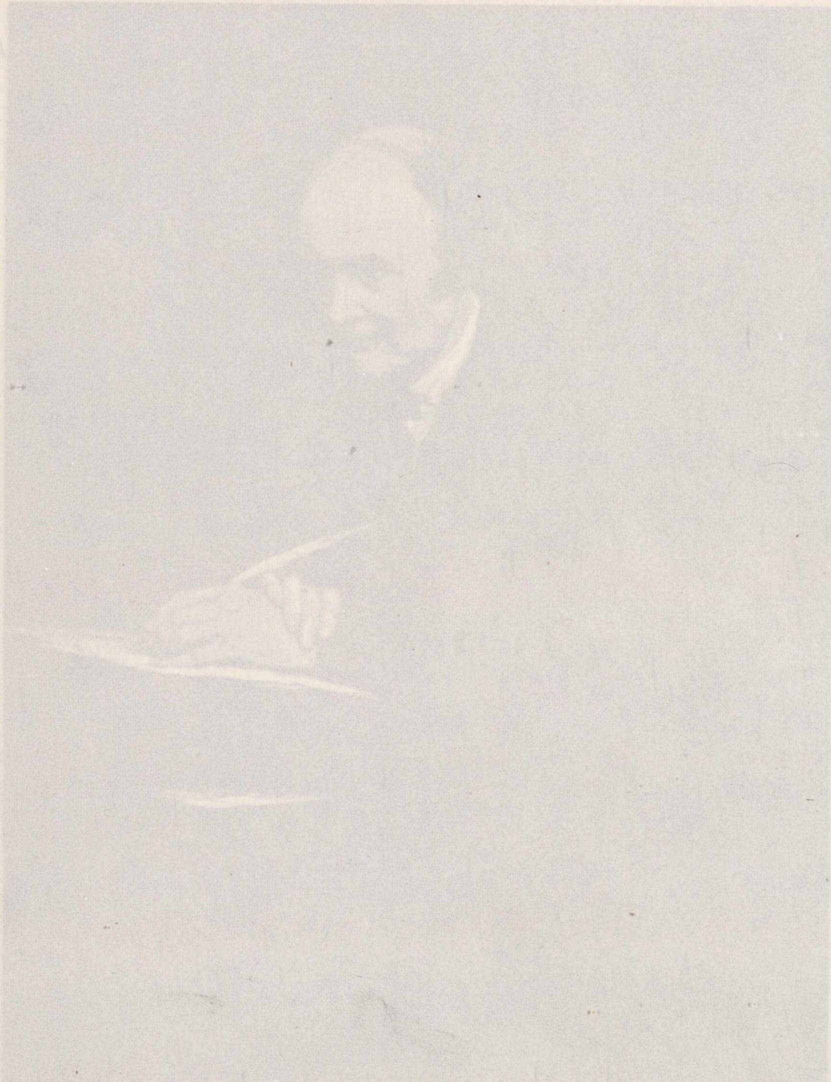
CAYLEY

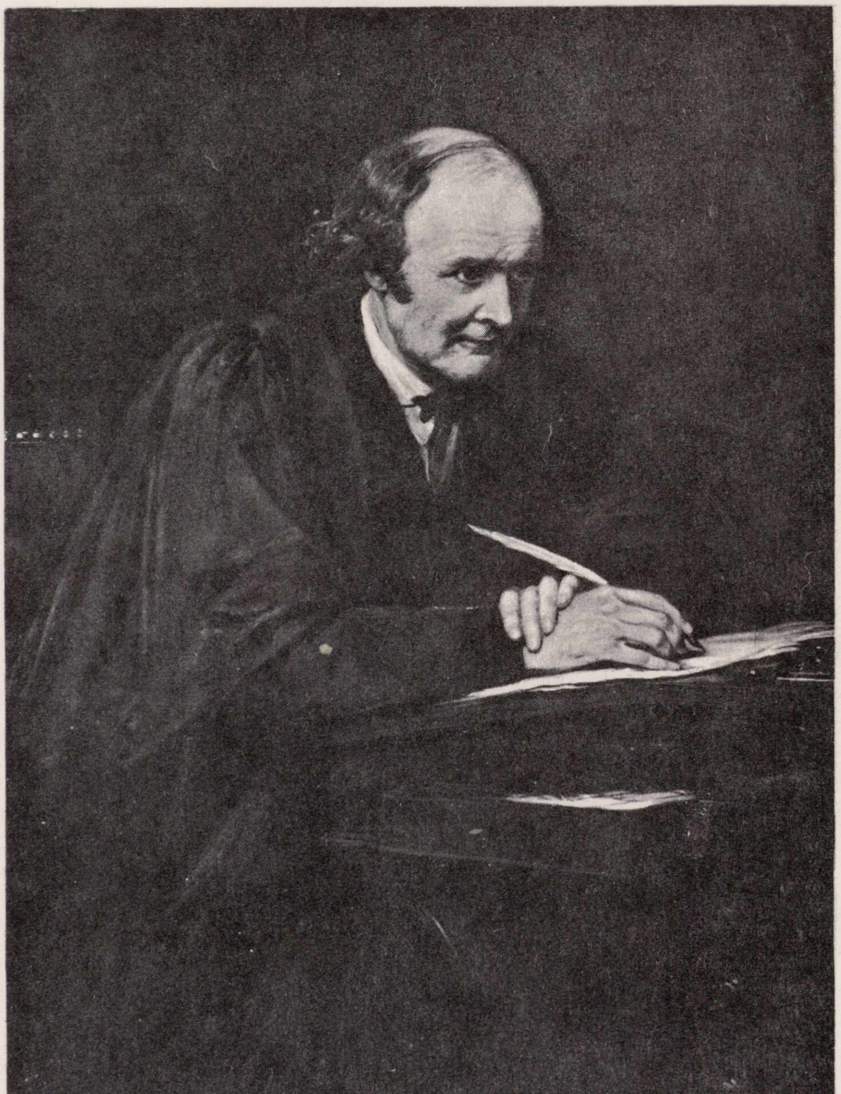
MATHEMATICAL PAPERS

6

1893

MATHEMATICAL PAPERS.





THE COLLECTED

MATHEMATICAL PAPERS

OF

ARTHUR CAYLEY, Sc.D., F.R.S.,

SADLERIAN PROFESSOR OF PURE MATHEMATICS IN THE UNIVERSITY OF CAMBRIDGE.

VOL. VI.

CAMBRIDGE:

AT THE UNIVERSITY PRESS.

1893

JOHNSON REPRINT CORPORATION
111 Fifth Avenue, New York 3, N.Y.

JOHNSON REPRINT COMPANY LIMITED
Berkeley Square House, London, W. 1.

First Reprinting, 1963, Johnson Reprint Corporation

CONTENTS.

CLASSIFICATION.

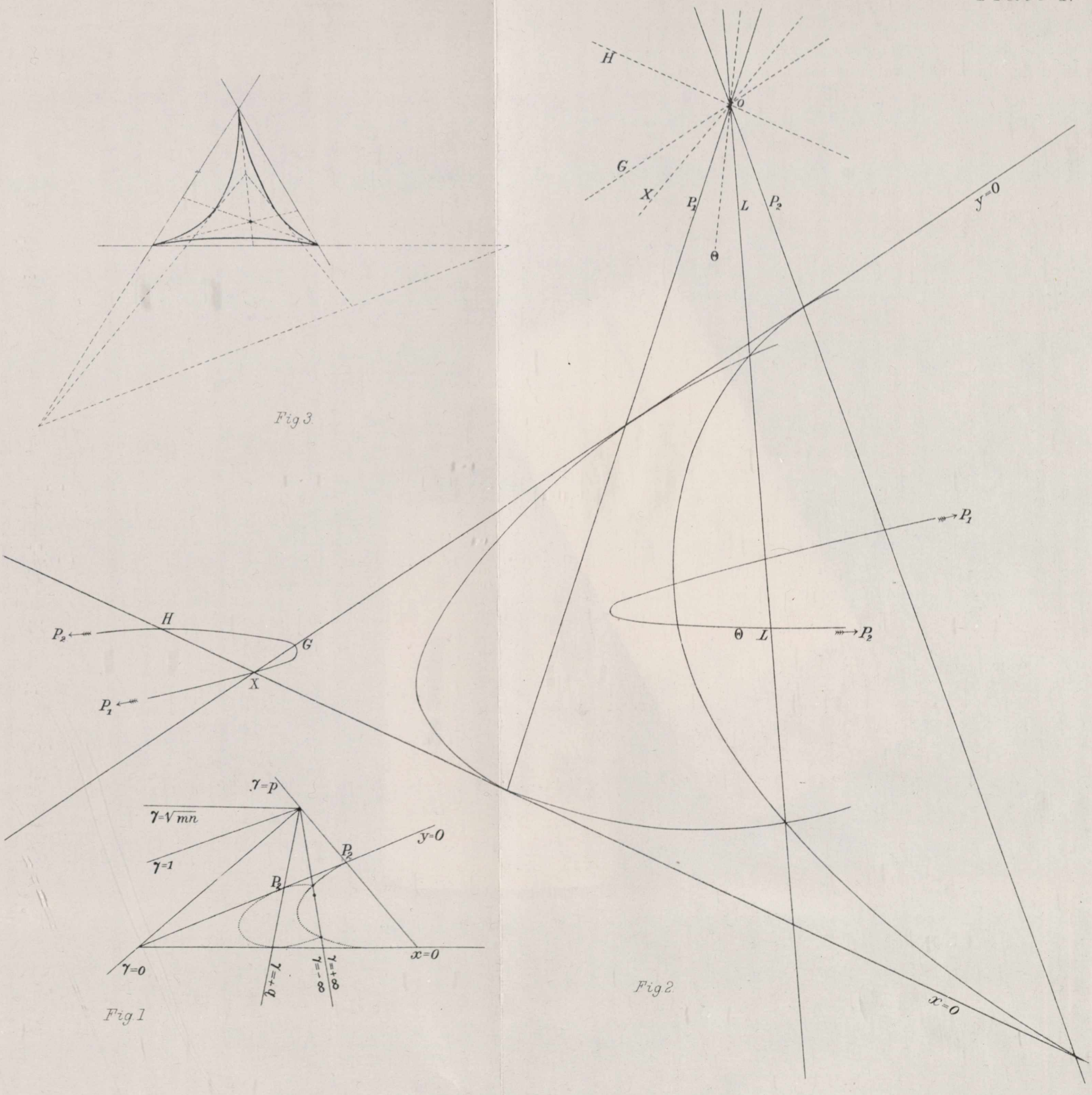


Fig. 3.

Fig. 2.

Fig. 1.

To face p. 52.

	AB	AE	AG	AH	AI	AJ	AM	AN	BC	BF	BH	BI	BJ	BN	BO	CD	CF	CG	CI	CJ	CK	CO	DE	DF	DG	DH	DJ	DK	DL	EF
AB		p	p	h	g	h	p	h	p	p	h	g	h	h	p	r	t	p	z	z	r	t	r	r	r	t	t	m	m	p
AE	p		h	g	h	p	h	p	r	p	z	z	t	t	r	r	r	t	t	r	m	m	p	t	z	z	p	r	t	p
AG	p	h		p	h	g	h	p	p	r	t	z	z	t	r	t	r	p	z	z	t	r	z	r	p	p	z	t	r	t
AH	h	g	p		p	h	p	h	t	z	h	z	w	w	t	r	r	r	r	t	m	m	z	t	p	p	z	r	t	z
AI	g	h	h	p		p	h	p	z	z	z	g	z	z	z	r	t	z	p	p	r	t	t	r	t	r	r	m	m	z
AJ	h	p	g	h	p		p	h	z	t	w	z	h	w	t	t	r	z	p	p	t	r	p	r	z	z	p	t	r	r
AM	p	h	h	p	h	p		g	r	r	t	z	t	z	p	m	m	t	t	r	r	r	t	m	t	r	r	r	r	t
AN	h	p	p	h	p	h	g		t	t	w	z	w	h	z	m	m	r	r	t	r	r	r	m	r	t	t	r	r	r
BC	p	r	p	t	z	z	r	t		h	p	h	g	p	h	p	h	p	h	g	p	h	r	z	t	p	z	t	r	t
BF	p	p	r	z	z	t	r	t	h		g	h	p	p	h	z	h	t	w	z	t	w	t	p	r	z	p	r	t	p
BH	h	z	t	h	z	w	t	w	p	g		p	h	h	p	p	z	r	t	z	r	t	t	z	r	p	z	r	t	z
BI	g	z	z	z	g	z	z	z	h	h	p		p	p	h	t	w	z	h	z	t	w	r	t	r	r	r	m	m	z
BJ	h	t	z	w	z	h	t	w	g	p	h	p		h	p	z	z	z	z	g	z	z	r	p	t	z	p	t	r	r
BN	h	t	t	w	z	w	z	h	p	p	h	p	h		g	r	t	r	t	z	p	z	m	r	m	t	t	r	r	r
BO	p	r	r	t	z	t	p	z	h	h	p	h	p	g		t	w	t	w	z	z	h	m	t	m	r	r	r	r	t
CD	r	r	t	r	r	t	m	m	p	z	p	t	z	r	t		g	h	p	h	h	p	p	g	h	p	h	h	p	z
CF	t	r	r	r	t	r	m	m	h	h	z	w	z	t	w	g		p	h	p	p	h	z	g	z	z	z	z	z	p
CG	p	t	p	r	z	z	t	r	p	t	r	z	z	r	t	h	p		g	h	h	p	z	z	h	t	w	w	t	p
CI	z	t	z	r	p	p	t	r	h	w	t	h	z	t	w	p	h	g		p	p	h	p	z	z	r	t	t	r	z
CJ	z	r	z	t	p	p	r	t	g	z	z	z	g	z	z	h	p	h	p		h	p	t	z	w	z	h	w	t	r
CK	r	m	t	m	r	t	r	r	p	t	r	t	z	p	z	h	p	h	p	h		g	t	z	w	t	w	h	z	r
CO	t	m	r	m	t	r	r	r	h	w	t	w	z	z	h	p	h	p	h	p	g		r	z	t	r	t	z	p	t
DE	r	p	z	z	t	p	t	r	r	t	t	r	r	m	m	p	z	z	p	t	t	r		h	g	h	p	p	h	h
DF	r	t	r	t	r	r	m	m	z	p	z	t	p	r	t	g	g	z	z	z	z	z	h		p	h	p	p	h	h
DG	r	z	p	p	t	z	t	r	t	r	r	r	t	m	m	h	z	h	z	w	w	t	g	p		p	h	h	p	z
DH	t	z	p	p	r	z	r	t	p	z	p	r	z	t	r	p	z	t	r	z	t	r	h	h	p		g	p	h	w
DJ	t	p	z	z	r	p	r	t	z	p	z	r	p	t	r	h	z	w	t	h	w	t	p	h	h	g		h	p	t
DK	m	r	t	r	m	t	r	r	t	r	r	m	t	r	r	h	z	w	t	w	h	z	p	p	h	p	h		g	t
DL	m	t	r	t	m	r	r	r	r	t	t	m	r	r	r	p	z	t	r	t	z	p	h	h	p	h	p	g		w
EF	p	p	t	z	z	r	t	r	t	p	z	z	r	r	t	z	p	p	z	r	r	t	h	h	z	w	t	t	w	
EG	t	h	h	z	w	z	w	t	r	r	r	t	r	m	m	z	p	p	z	t	t	r	g	z	g	z	z	z	z	p
EH	z	g	z	g	z	z	z	z	r	z	p	p	t	t	r	t	t	r	r	r	m	m	h	w	z	h	z	t	w	h
EI	z	h	w	z	h	t	w	t	t	z	p	p	r	r	t	p	z	z	p	r	r	t	p	z	z	t	r	r	t	g
EL	r	p	t	z	t	r	z	p	m	t	t	r	m	r	r	t	t	r	r	m	r	r	h	w	z	w	t	z	h	h
EM	t	h	w	z	w	t	h	z	m	r	r	t	m	r	r	r	r	t	t	m	r	r	p	t	z	t	r	p	z	p
FI	z	z	t	p	p	r	t	r	w	h	z	h	t	t	w	z	h	z	h	t	t	w	z	p	p	t	r	r	t	g
FH	z	z	r	p	p	t	r	t	z	g	g	z	z	z	z	z	p	r	t	p	r	t	w	h	t	h	z	t	w	h
FL	r	t	m	t	r	m	r	r	t	p	z	t	r	p	z	z	p	r	t	r	p	z	w	h	t	w	t	z	h	h
FO	t	r	m	r	t	m	r	r	w	h	z	w	t	z	h	z	h	t	w	t	z	h	t	p	r	t	r	p	z	p
GI	z	w	h	t	h	z	w	t	z	t	r	p	p	r	t	z	z	g	g	z	z	z	z	p	p	r	t	t	r	z
GJ	z	z	g	z	z	g	z	z	z	r	t	p	p	t	r	w	t	h	z	h	w	t	z	t	h	z	h	w	t	r
GK	r	t	p	r	t	z	z	p	t	m	m	r	t	r	r	w	t	h	z	w	h	z	z	t	h	t	w	h	z	r
GM	t	w	h	t	w	z	h	z	r	m	m	t	r	r	r	t	r	p	z	t	z	p	z	r	p	r	t	z	p	t
HJ	w	z	z	h	t	h	t	w	z	z	h	t	h	w	t	z	p	t	r	p	t	r	z	z	z	g	g	z	z	t
HL	t	z	r	p	r	t	p	z	r	z	p	r	t	z	p	t	t	m	m	r	r	r	w	w	t	h	z	z	h	w
HN	w	z	t	h	t	w	z	h	t	z	h	t	w	h	z	r	r	m	m	t	r	r	t	t	r	p	z	p	z	t
IM	z	w	w	t	h	t	h	z	t	t	r	p	r	p	z	r	t	z	p	r	p	z	t	r	t	m	t	r	r	z
IO	z	t	t	r	p	r	z	p	w	w	t	h	t	z	h	t	w	z	h	t	z	h	r	t	r	m	m	r	r	z
JK	t	r	z	t	r	p	p	z	z	r	t	r	p	z	p	w	t	w	t	h	h	z	t	t	w	z	h	h	z	m
JN	w	t	z	w	t	h	z	h	z	t	w	t	h	h	z	t	r	t	r	p	z	p	r	r	t	z	p	z	p	m
KL	m	r	r	r	m	r	t	t	r	r	r	m	r	t	t	z	p	t	r	t	p	z	z	z	z	z	z	g	g	t
KM	r	t	z	r	t	p	p	z	r	m	m	r	r	t	t	t	r	z	p	t	p	z	p	r	z	r	t	p	z	r
KN	t	r	p	t	r	z	z	p	p	r	t	r	z	p	z	t	r	t	r	z	p	z	r	r	t	p	z	p	z	m
KO	r	m	r	m	r	r	t	t	z	t	r	t	p	z	p	z	z	z	z	z	g	g	r	p	t	r	t	p	z	r
LM	r	z	t	p	t	r	p	z	m	r	r	r	m	t	t	r	r	r	r	m	t	t	z	t	p	t	r	z	p	z
LN	t	p	r	z	r	t	z	p	r	p	z	r	t	p	z	r	r	m	m	r	t	t	t	t	r	z	p	z	p	t
LO	r	r	m	r	r	m	t	t	t	z	p	t	r	z	p	p	z	r	t	r	z	p	t	z	r	t	r	z	p	z
MN	z	z	z	z	z	z	g	g	r	r	t	p	t	p	z	m	m	r	r	r	t	t	r	m	r	r	r	t	t	r
MO	p	t	t	r	z	r	p	z	t	t	r	z	r	z	p	r	t	p	z	r	z	p	r	r	r	m	m	t	t	p
NO	z	r	r	t	p	t	z	p	z	z	z	z	z	g	g	r	t	r	t	p	z	p	m	r	m	r	r	t	t	r

AB AE AG AH AI AJ AM AN BC BF BH BI BJ BN BO CD CF CG CI CJ CK CO DE DF DG DH DJ DK DL EF

EG	EH	EI	EL	EM	FI	FH	FL	FO	GI	GJ	GK	GM	HJ	HL	HN	IM	IO	JK	JN	KL	KM	KN	KO	LM	LN	LO	MN	MO	NO	
t	z	z	r	t	z	z	r	t	z	x	r	t	w	t	w	z	z	t	w	m	r	t	r	r	t	r	z	p	z	AB
h	g	h	p	h	x	x	t	r	w	z	t	w	z	z	z	w	t	r	t	r	t	r	m	z	p	r	z	t	r	AE
h	x	w	t	w	t	r	m	m	h	g	p	h	z	r	t	w	t	z	z	r	z	p	r	t	r	m	z	t	r	AG
z	g	x	z	x	p	p	t	r	t	z	r	t	h	p	h	t	r	t	w	r	r	t	m	p	z	r	z	r	t	AH
w	x	h	t	w	p	p	r	t	h	z	t	w	t	r	t	h	p	r	t	m	t	r	r	t	r	r	z	z	p	AI
z	z	t	r	t	r	t	m	m	z	g	z	z	h	t	w	t	r	p	h	r	p	z	r	r	t	m	z	r	t	AJ
w	z	w	z	h	t	r	r	r	w	z	z	h	t	p	x	h	x	p	z	t	p	z	t	p	z	t	g	p	x	AM
t	z	t	p	z	r	t	r	r	t	z	p	z	w	z	h	z	p	z	h	t	z	p	t	z	p	t	g	z	p	AN
r	r	t	m	m	w	z	t	w	z	z	t	r	z	r	t	t	w	z	z	r	r	p	z	m	r	t	r	t	z	BC
r	z	z	t	r	h	g	p	h	t	r	m	m	x	z	z	t	w	r	t	r	m	r	t	r	p	z	r	t	z	BF
r	p	p	t	r	z	g	z	z	r	t	m	m	h	p	h	r	t	t	w	r	m	t	r	r	x	p	t	r	z	BH
t	p	p	r	t	h	z	t	w	p	p	r	t	t	r	t	p	h	r	t	m	r	r	t	r	r	t	p	z	z	BI
r	t	r	m	m	t	z	r	t	p	p	t	r	h	t	w	r	t	p	h	r	r	z	p	m	t	r	t	r	z	BJ
m	t	r	r	r	t	z	p	x	r	t	r	r	w	z	h	p	z	z	h	t	t	p	z	t	p	z	p	z	g	BN
m	r	t	r	r	w	z	z	h	t	r	r	r	t	p	z	z	h	p	z	t	t	z	p	t	z	p	z	p	g	BO
z	t	p	t	r	z	x	z	z	z	w	w	t	z	t	r	r	t	w	t	z	t	t	z	r	r	p	m	r	r	CD
p	t	z	t	r	h	p	p	h	z	t	t	r	p	t	r	t	w	t	r	p	r	r	x	r	r	z	m	t	t	CF
p	r	z	r	t	z	r	r	t	g	h	h	p	t	m	m	z	z	w	t	t	z	t	z	r	m	r	r	p	r	CG
z	r	p	r	t	h	t	t	w	g	z	z	z	r	m	m	p	h	t	r	r	p	r	z	r	m	t	r	z	t	CI
t	r	r	m	m	t	p	r	t	z	h	w	t	p	r	t	r	t	h	p	t	t	z	z	m	r	r	r	r	p	CJ
t	m	r	r	r	t	r	p	z	z	w	h	x	t	r	r	p	z	h	z	p	p	p	g	t	t	z	t	z	x	CK
r	m	t	r	r	w	t	z	h	z	t	z	p	r	r	r	x	h	z	p	z	z	z	g	t	t	p	t	p	p	CO
g	h	p	h	p	x	w	w	t	z	z	z	z	z	w	t	t	r	t	r	z	p	r	r	x	t	t	r	r	m	DE
z	w	z	w	t	p	h	h	p	p	t	t	r	z	w	t	r	t	t	r	z	r	r	p	t	t	z	m	r	r	DF
g	z	z	z	z	p	t	t	r	p	h	h	p	z	t	r	t	r	w	t	z	z	t	t	p	r	r	r	r	m	DG
z	h	t	w	t	t	h	w	t	r	z	t	r	g	h	p	m	m	z	z	z	r	p	r	t	z	t	r	m	r	DH
x	z	r	t	r	r	z	t	r	t	h	w	t	g	z	z	t	m	h	p	z	t	z	t	r	p	r	r	m	r	DJ
z	t	r	x	p	r	t	z	p	t	w	h	x	z	z	p	r	r	h	z	g	p	p	p	z	x	z	t	t	t	DK
z	w	t	h	x	t	w	h	z	r	t	x	p	z	h	z	r	r	z	p	g	z	z	z	p	p	p	t	t	t	DL
p	h	g	h	p	g	h	h	p	z	r	r	t	t	w	t	z	z	m	m	t	r	m	r	x	t	z	r	p	r	EF
	p	h	p	h	z	t	t	r	h	p	p	h	p	t	r	w	t	t	r	p	z	r	r	z	r	r	t	t	m	EG
p		p	h	p	z	h	w	t	t	p	r	t	p	h	p	t	r	r	t	t	r	r	m	x	z	t	p	r	r	EH
h	p		p	h	g	z	z	z	h	t	t	w	r	t	r	h	p	m	m	r	t	m	r	z	r	p	t	z	r	EI
p	h	p		g	z	w	h	z	t	r	p	z	t	h	z	z	p	r	r	p	z	t	t	g	p	p	z	z	t	EL
h	p	h	g		z	t	z	p	w	t	z	h	r	z	p	h	x	r	r	z	p	t	t	g	z	z	p	p	t	EM
z	z	g	z	z		p	p	h	p	r	r	t	r	t	r	p	h	m	m	r	r	m	t	p	r	z	r	z	t	FI
t	h	z	w	t	p		h	p	r	r	m	m	p	h	p	r	t	r	t	t	m	r	r	t	z	z	r	r	p	FH
t	w	z	h	z	p	h		g	r	m	r	r	t	h	z	p	z	r	r	p	t	t	z	p	p	g	t	z	z	FL
r	t	z	z	p	h	p	g		t	m	r	r	r	z	p	z	h	r	r	z	t	t	p	z	z	g	t	p	p	FO
h	t	h	t	w	p	r	r	t		p	p	h	r	m	m	h	p	t	r	r	z	r	p	t	m	r	t	z	r	GI
p	p	t	r	t	r	r	m	m	p		h	p	p	r	t	m	r	h	p	t	z	z	t	r	r	m	p	r	r	GJ
p	r	t	p	z	r	m	r	r	p	h		g	t	r	r	z	p	h	z	p	g	p	p	z	t	t	z	z	t	GK
h	t	w	z	h	t	m	r	r	h	p	g		r	r	r	h	z	z	p	z	g	z	z	p	t	t	p	p	t	GM
p	p	r	t	r	r	p	t	r	r	p	t	r		p	h	m	m	p	h	p	r	z	r	r	z	r	t	m	t	HJ
t	h	t	h	z	t	h	h	z	m	r	r	r	p		g	r	r	p	z	p	t	z	t	p	g	p	z	t	z	HL
r	p	r	z	p	r	p	z	p	m	t	r	r	h	g		r	r	z	h	z	t	p	t	z	g	z	p	t	p	HN
w	t	h	z	h	p	r	p	z	h	m	z	h	m	r	r		g	r	r	t	p	t	z	p	t	z	p	g	z	IM
t	r	p	p	z	h	t	z	h	p	r	p	z	m	r	r	g		r	r	t	z	t	p	z	t	p	z	g	p	IO
t	r	m	r	r	m	r	r	r	t	h	h	z	p	p	z	r	r		g	p	p	g	p	t	z	t	z	t	z	JK
r	t	m	r	r	m	t	r	r	r	p	x	p	h	z	h	r	r	g		z	z	g	z	t	p	t	p	t	p	JN
p	t	r	p	x	r	t	p	z	r	t	p	z	p	p	x	t	t	p	x		h	h	h	h	h	h	w	w	w	KL
z	r	t	z	p	r	m	t	t	z	z	g	g	r	t	t	p	z	p	z	h		h	h	h	w	w	h	h	w	KM
r	r	m	t	t	m	r	t	t	r	z	p	z	z	z	p	t	t	g	g	h	h		h	w	h	w	h	w	h	KN
r	m	r	t	t	t	r	z	p	p	t	p	z	r	t	t	z	p	p	z	h	h	h		w	w	h	w	h	h	KO
z	z	z	g	g	p	t	p	z	t	r	x	p	r	p	z	p	x	t	t	h	h	w	w		h	h	h	h	w	LM
r	z	r	p	x	r	x	p	z	m	r	t	t	x	g	g	t	t	z	p	h	w	h	w	h		h	h	w	h	LN
r	t	p	p	z	z	z	g	g	r	m	t	t	r	p	z	z	p	t	t	h	w	w	h	h	h		w	h	h	LO
t	p	t	z	p	r	r	t	t	t	p	z	p	t	z	p	p	z	z	p	w	h	h	w	h	h	w		h	h	MN
t	r	z	z	p	x	r	x	p	z	r	z	p	m	t	t	g	g	t	t	w	h	w	h	h	w	h	h		h	MO
m	r	r	t	t	t	p	z	p	r	r	t	t	t	z	p	z	p	z	p	w	w	h	h	w	h	h	h	h		NO

EG EN EI EL EM FI FH FL FO GI GJ GN GM HJ HL HN IM IO JK JN KL KM KN KO LM LN LO MN MO NO

Fig 1.

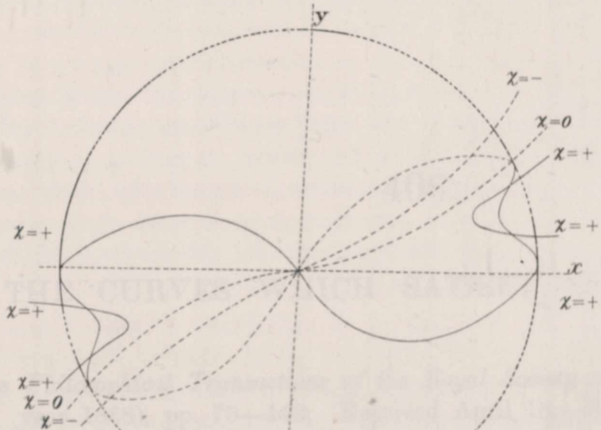


Fig. 2.

The lower cusp of the Bicorn is drawn out of its true position which is much further off along the asymptote, the co-ordinates in fact are $x=-76\frac{22}{27}$, $y=-44\frac{1}{3}$ *(the co-ordinates of the upper or node-cusp being* −1, 1.

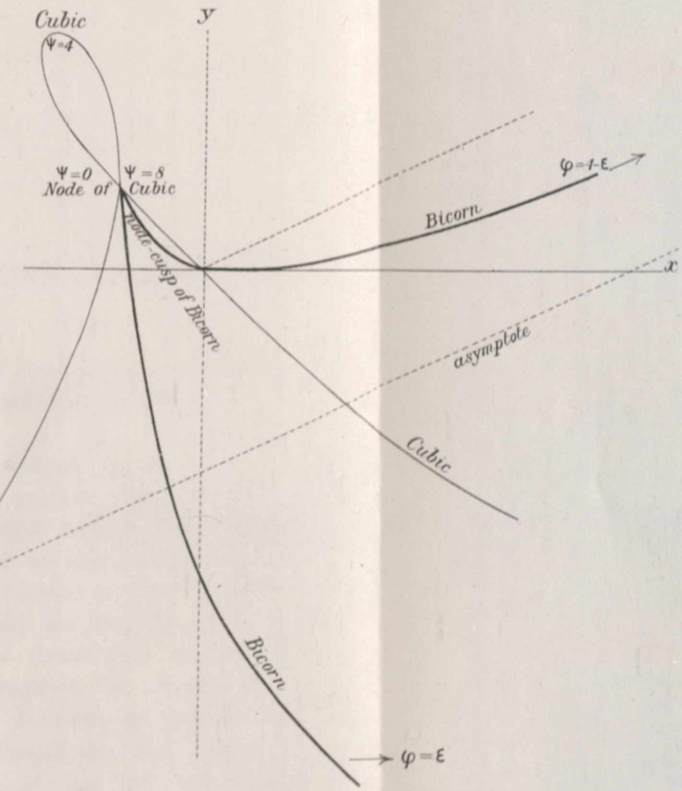


Fig. 4.

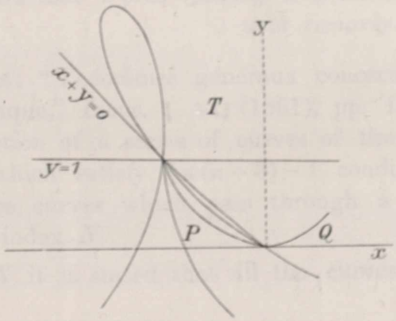
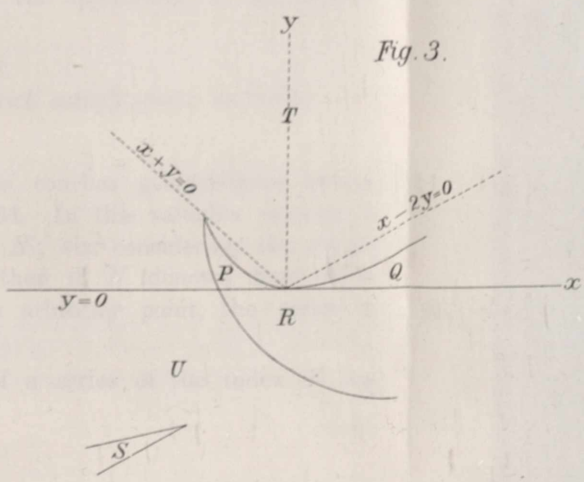


Fig. 3.



thereof, as regards the line-singularities of the inflexion and the double tangent. We are thus led to consider as ordinary singularities in the theory the above-mentioned four singularities of the inflexion, the double tangent, the node and the cusp; and we know further that any other singularity whatever of a plane curve is compounded (in a definite manner) of a certain number of some or all of these singularities.

But in the theory of surfaces, starting in like manner with the general equation $(x, y, z, w)^n = 0$, such a surface has these singularities, the node-couple torse, and the spinode-torse; each of these is in general an indecomposable torse of a certain kind (but there is the new cause of complication that it may break into two or more separate torses), but we do not know the analytical expression of these singularities, nor consequently the analytical expression of the curve-singularities which correspond to them, the nodal curve and the cuspidal curve. Thus if we attempt to start with a surface $(x, y, z, w)^n = 0$ having a nodal curve, we can indeed write down the equation in its most general form, viz. if the nodal curve has for its complete expression the k equations $P = 0$, $Q = 0$, $R = 0$, &c. (viz. if the curve is such that every surface whatever through the curve is of the form $\Pi_1 = AP + BQ + CR + \ldots = 0$) then the most general equation of the surface having this curve as a nodal curve is $(A, B, C, \ldots)(P, Q, R, \ldots)^2 = 0$, but this form is far too complicated to be worked with; and if for simplicity we take the nodal curve to be a complete intersection $P = 0$, $Q = 0$ and consequently the equation of the surface to be $(A, B, C)(P, Q)^2 = 0$, then it is by no means clear that we do not in this way introduce limitations extraneous to the general theory. The same difficulty applies (if rather) and with yet greater force, to the cuspidal curve; and even if we could deal separately with the cases of a surface having a given nodal curve, and a given cuspidal curve, this would in no wise solve the problem for the more general case of a surface having a given nodal curve and a given cuspidal curve. It is to be added that the general surface of the order n has no plane- or point-singularities, and thus that such singularities (which correspond most nearly to the singularities considered in the theory of reciprocal curves) present themselves in the theory of reciprocal surfaces as extraordinary singularities.

END OF VOL. VI.