185.

NOTE ON THE 'CIRCULAR RELATION' OF PROF. MÖBIUS.

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THEOREM.

Given the points

$$A,\ B,\ C\,;\ P,$$

and the points

$$A',\ B',\ C'\,;$$

describe the circles α, β, γ, ω as follows: viz.

$$\begin{array}{lll} \alpha & \text{through} & (B,\ C,\ P), \\ \beta & \quad ,, & (C,\ A,\ P), \\ \gamma & \quad ,, & (A,\ B,\ P), \\ \omega & \quad ,, & (A,\ B,\ C), \end{array}$$

and the circles α', β', γ', ω' as follows: viz.,

$$\begin{array}{lllllllll} \alpha' & \text{through} & (B',\ C') & \text{cutting} & \omega' & \text{at the angle at which} & \alpha & \text{cuts} & \omega, \\ \beta' & \quad ,, & (C',\ A') & \quad ,, & \omega' & \quad ,, & \beta & \quad ,, & \omega, \\ \gamma' & \quad ,, & (A',\ B') & \quad ,, & \omega' & \quad ,, & \gamma & \quad ,, & \omega, \text{ and} \\ \omega' & \quad ,, & (A',\ B',\ C') & & & & & & \end{array}$$

then will α', β', γ' meet in a point P', i.e. we shall have the points A', B', C'; P'

such that the circles α', β', γ', ω' pass

$$\begin{array}{lll} \alpha' & \text{through} & (B',\ C',\ P'), \\ \beta' & ,, & (C',\ A',\ P'), \\ \gamma' & ,, & (A',\ B',\ P'), \\ \omega' & ,, & (A',\ B',\ C'). \end{array}$$

We may construct in this manner two figures, such that to three points of the first figure there correspond in the second figure three points which may be taken at pleasure, but these once selected to every other point of the first figure there will correspond in the second figure a perfectly determinate point. And the two figures will be such that whenever in the first figure four or more points lie in a circle, then in the second figure the corresponding points will also lie in a circle. The relation in question is due to Prof. Möbius, who has termed it the *Kreis-verwandschaft* (circular relation) of two plane figures. See his paper *Crelle*, t. LII. [1856], pp. 218—228, extracted from the Berichte of the *Royal Saxon Society of Sciences* of the 5th Feb. 1853, [and *Werke*, t. II. pp. 207—217].