13.

ON THE LIMITS TO THE ORDER AND DEGREE OF THE FUNDAMENTAL INVARIANTS OF BINARY QUANTICS.

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THE developments which I have recently given to Professor Cayley's second method of dealing with invariants (the first method being that which has been exclusively used by Professor Gordan), has led me through the theory of the Canonical Generating Fraction to the following results, showing that the degree and order of the fundamental invariants and covariants to a quantic or system of quantics are subject to algebraical limits of a very simple kind, and I think it right that these results should not be withheld from the knowledge of those who are pursuing another and, as it seems to me, much more arduous and less promising direction of inquiry into the same subject.

By order I mean the dimensions of a derived form in the coefficients of its primitive (Clebsch and Gordan's *grad*), and by degree the dimensions in the variables (Clebsch and Gordan's *ordnung*).

First as to degree.

If there be a system of n, n', n''.. odd degreed quantics and ν, ν', .. &c., even ones, then (with the exception of the case when the system reduces to a single linear function or a single quadratic) the degree of any irreducible covariant to the system has for a superior limit $\Sigma\left(\frac{n^2+1}{2}\right)+\Sigma\left(\frac{\nu^2}{2}\right)-2$.

Thus, for example, where there is but one quantic, the limit is $\frac{n^2-3}{2}$ or $\frac{\nu^2-4}{2}$, according as the degree is n odd or ν even.

Secondly, as to order.

As the expressions become somewhat complicated when there are several quantics, I shall confine myself to a statement applicable to a single quantic, distinguishing between the three cases when n (its degree) is evenly even, oddly even, and odd.

A. When n contains 4, the superior limits for the order of the invariants and covariants respectively are for the former $\frac{(n+1)(n-4)}{2}$, and for the latter $\frac{(n+2)(n-3)}{2}$.

B. When n is even, but not divisible by 4, and is greater than 2, the limits for the two species are $\frac{3n^2-6n-12}{4}$ and $\frac{(n+2)(3n-8)}{4}$ respectively.

C. When n is any odd number greater than 3, the order of the invariants has for its limit $\frac{3}{2}(n+1)(n-3)$, and when it is any odd number greater than unity, the order of the covariants has for its limit $\frac{3n^2-4n-9}{2}$.

Further investigations will, I have good reason to believe, lead to considerably lower limits than those given for cases *B* and *C*.

Although morally certain, the three formulæ *A*, *B*, *C* cannot be considered at present apodictically established; the formula respecting the limit to *degree* may, I believe, be regarded as admitting of a complete demonstration. There exists, however, a superior limit to the orders of the fundamental invariants or covariants, which may be regarded as subject to direct demonstration even in our present state of knowledge; this when n is even is n^2-2n-3 for invariants, and n^2-n-4 for covariants; and when n is odd, the corresponding limits are $2n^2-3n-5$ for invariants, and $2n^2-2n-5$ for covariants. But I have no moral doubt whatever of the validity of the formulæ *B* and *C* as they stand, and next to none of the validity of formula *A*.