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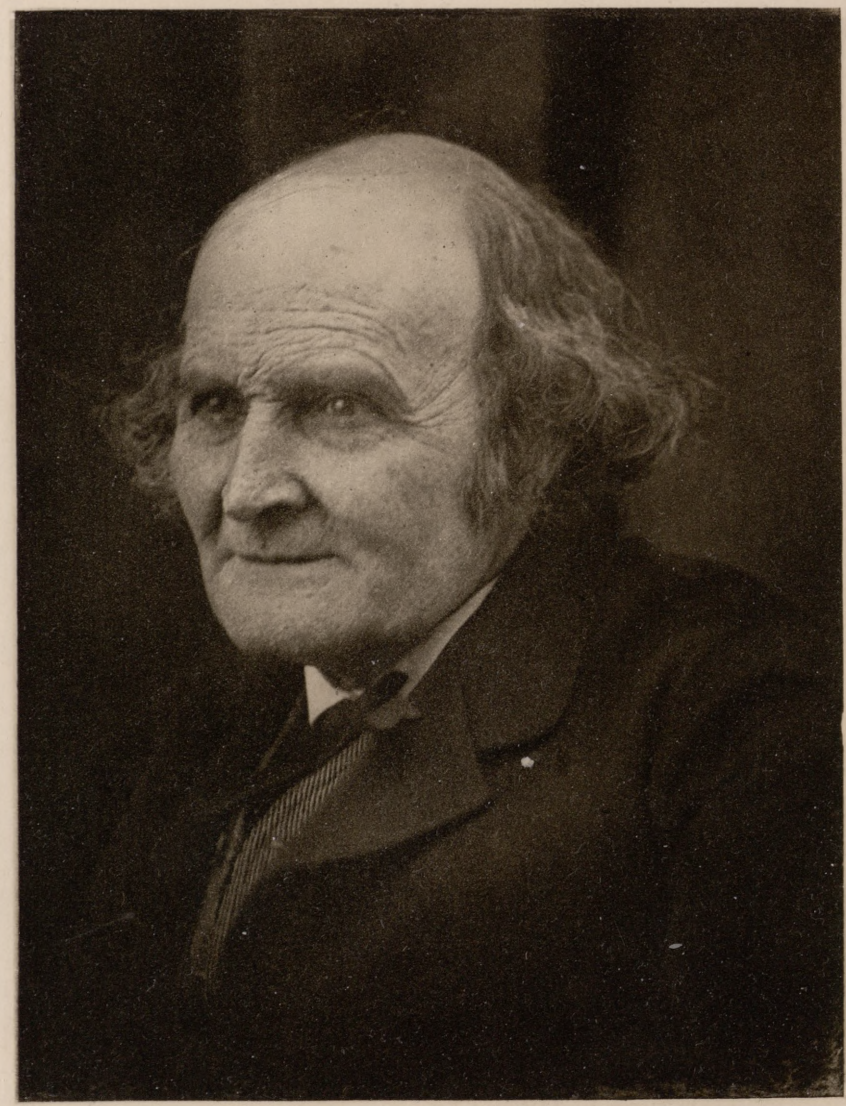

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THE COLLECTED

MATHEMATICAL PAPERS

OF

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THE present volume contains 93 papers numbered 706 to 798, published, with the exception of one series, for the most part in the years 1878 to 1883. This series is constituted by the articles which Professor Cayley wrote for the *Encyclopædia Britannica* between the years 1878 and 1888; it seemed desirable to place these together in the same volume, in spite of the departure from the chronological arrangement which governs the sequence of the papers in the volumes generally. The Syndics of the University Press desire to acknowledge their obligation to Messrs Adam and Charles Black, Publishers of the ninth Edition of the *Encyclopædia Britannica*, for their courteous consent in allowing these articles to be included in the *Collected Mathematical Papers*. Exact references to the volumes, from which the articles are extracted, will be found in the Table of Contents.

The frontispiece to the present volume is a reproduction by Mr A. G. Dew-Smith, of Trinity College, of a photograph of Professor Cayley which he made in the year 1885. The Syndics of the Press desire to acknowledge their obligation to Mr Dew-Smith.

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A. R. FORSYTH.

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