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TABLES FOR THE BINARY SEXTIC.

THE LEADING COEFFICIENTS OF THE FIRST 18 OF THE 26 COVARIANTS.

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INCLUDING the sextic itself, the number of covariants of the binary sextic is $=26$, as shown in the table p. 296 of Clebsch's *Theorie der binären algebraischen Formen*, Leipzig, 1872; viz. this is

	Order						
Deg.	0	2	4	6	8	10	12
1				f			
2	A		i		H		
3		l		p	(f, i)		T
4	B		$(f, l)_2$	(f, l)		(H, i)	
5		$(i, l)_2$	(i, l)		(H, l)		
6	A_{ll}			(p, l) $((f, i), l)_2$			
7		$(f, l^2)_4$	$(f, l^2)_3$				
8		$(i, l^2)_3$					
9			$((f, i), l^2)_4$				
10	$(f, l^3)_6$	$(f, l^3)_5$					
12		$((f, i), l^3)_6$					
15	$((f, i), l^4)_8$						

Or, using the capital letters $A, B, \ldots, Z$ to denote the 26 covariants in the same order, the table is

	0	2	4	6	8	10	12
1				A			
2	B		C		D		
3		E		F	$G, =(A, C)^1$		H
4	I		$J, =(A, E)^2$	$K, =(A, E)^1$		$L, =(D, C)^1$	
5		$M, =(C, E)^2$	$N, =(C, E)^1$		$O, =(D, E)^1$		
6	P			$Q, =(F, E)^1$ $R, =(G, E)^2$			
7		$S, =(A, E^2)^4$	$T, =(A, E^2)^3$				
8		$U, =(C, E^2)^3$					
9			$V, =(G, E^2)^4$				
10	$W, =(A, E^3)^6$	$X, =(A, E^3)^5$					
12		$Y, =(G, E^3)^6$					
15	$Z, =(G, E^4)^8$						

A is the sextic.
B is Salmon's A, p. 202.
I ,, ,, B, p. 203.
P is Salmon's C, p. 204.
W ,, ,, D, p. 207.
Z ,, ,, E, p. 253.
The references are to Salmon's *Higher Algebra*, 2nd Ed., 1866.

In the present short paper I give the leading coefficients of the first 18 covariants, A to R (some of these are of course known values, but it is convenient to include them): for the next four covariants S, T, U, V, the leading coefficients depend upon the coefficients of A, C, G and E^2, viz. writing

$$\begin{aligned} A &= (a,\ b,\ c,\ d,\ e,\ f,\ g \between x, y)^6, \\ E^2 &= (\alpha,\ \tfrac{1}{4}\beta,\ \tfrac{1}{6}\gamma,\ \tfrac{1}{4}\delta,\ \epsilon \between x, y)^4, \\ C &= (\alpha',\ \tfrac{1}{4}\beta',\ \tfrac{1}{6}\gamma',\ \tfrac{1}{4}\delta',\ \epsilon' \between x, y)^4, \\ G &= (\alpha'',\ \tfrac{1}{8}\beta'',\ \tfrac{1}{28}\gamma'',\ \tfrac{1}{56}\delta'',\ \tfrac{1}{70}\epsilon'', \ldots \between x, y)^8, \end{aligned}$$

we have

$$\begin{aligned} &S, \text{ Coeff. } x^2 = a\epsilon - b\delta + c\gamma - d\beta + e\alpha, \\ &T, \quad ,, \quad x^4 = a\delta - 2b\gamma + 3c\beta - 4d\alpha, \\ &U, \quad ,, \quad x^2 = 2\alpha'\delta - \beta'\gamma + \gamma'\beta - 2\delta'\alpha, \\ &V, \quad ,, \quad x^4 = 280\alpha''\epsilon - 35\beta''\delta + 10\gamma''\gamma - 20\delta''\beta + 24\epsilon''\alpha. \end{aligned}$$

Similarly the invariant W and the leading coefficients of X, Y depend on the coefficients of A, G and E^3; and the invariant Z depends on the coefficients of G and E^4.

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But these two invariants W and Z have been already calculated; viz. as already mentioned, W is Salmon's invariant D, and Z his invariant E, given each of them in the second edition of his *Higher Algebra* (but not reproduced in the third edition): on account of the great length of these expressions, it has been thought that it was not expedient to give them here.

For the reason appearing above, I have added the expressions for the remaining coefficients of C, E, G.

A, x^6	B, x^0	C, x^4	D, x^8	E, x^2	F, x^6	G, x^8	H, x^{12}
a + 1	$a\,g$ + 1	$a\,e$ + 1	$a\,c$ + 1	$a\,cg$ + 1	$a\,ce$ + 1	a^2f + 1	a^2d + 1
	a^0bf − 6	a^0bd − 4	a^0b^2 − 1	df − 3	d^2 − 1	$a\,be$ − 5	$a\,bc$ − 3
	ce + 15	c^2 + 3		e^2 + 2	a^0b^2e − 1	cd + 2	a^0b^3 + 2
	d^2 − 10			a^0b^2g − 1	bcd + 2	a^0b^2d + 8	
				bcf + 3	c^3 − 1	bc^2 − 6	
				bde − 1			
				c^2e − 3			
				cd^2 + 2			

I, x^0	J, x^4	K, x^6	L, x^{10}	M, x^2	N, x^4	O, x^8
$a\,ceg$ + 1	a^2f^2 + 1	a^2dg + 1	a^2cf + 1	a^2cg^2 + 1	a^2cfg − 1	a^2cdg 0
cf^2 − 1	$a\,bef$ − 10	ef − 1	de − 1	dfg − 6	deg + 1	cef − 1
d^2g − 1	cdf + 4	$a\,bcg$ − 3	$a\,b^2f$ − 1	e^2g + 8	df^2 + 3	d^2f + 3
def + 2	ce^2 + 16	bdf − 2	bce − 2	ef^2 − 3	e^2f − 3	de^2 − 2
e^3 − 1	d^2e − 12	be^2 + 5	bd^2 + 4	$a\,b^2g^2$ − 1	$a\,b^2fg$ + 1	$a\,b^2dg$ 0
a^0b^2eg − 1	a^0b^2df + 16	c^2f + 9	c^2d − 1	$bcfg$ + 6	$bceg$ + 2	b^2ef + 1
b^2f^2 + 1	b^2e^2 + 9	cde − 17	a^0b^3e + 3	$bdeg$ − 34	bcf^2 − 3	bc^2g 0
$bcdg$ + 2	bc^2f − 12	d^3 + 8	b^2cd − 6	bdf^2 + 48	bd^2g − 4	$bcdf$ − 14
$bcef$ − 2	$bcde$ − 76	a^0b^3g + 2	bc^3 + 3	be^2f − 18	$bdef$ − 12	bce^2 + 11
bd^2f − 2	bd^3 + 48	b^2cf − 6		c^2eg + 18	be^3 + 15	bd^2e + 1
bde^2 + 2	c^3e + 48	b^2de + 2		c^2f^2 − 45	c^2dg + 1	c^3f + 9
c^3g − 1	c^2d^2 − 32	bc^2e + 6		cd^2g + 4	c^2ef + 9	c^2de − 14
c^2df + 2		bcd^2 − 4		$cdef$ + 78	cd^2f + 4	cd^3 + 6
c^2e^2 + 1				ce^3 − 36	cde^2 − 21	a^0b^3cg 0
cd^2e − 3				d^3f − 48	d^3e + 8	b^3df + 8
d^4 + 1				d^2e^2 + 28	a^0b^3eg − 3	b^3e^2 − 9
				a^0b^2ceg 0	b^2cdg + 6	b^2c^2f − 6
				b^2d^2g + 64	b^2cef + 9	b^2cde + 16
				b^2def − 144	b^2d^2f + 32	b^2d^3 − 8
				b^2e^3 + 81	b^2de^2 − 39	bc^3e − 3
				bc^2dg − 96	bc^3g − 3	bc^2d^2 + 2
				bc^2ef + 108	bc^2df − 66	
				bcd^2f + 96	bc^2e^2 + 18	
				$bcde^2$ − 126	bcd^2e + 76	
				bd^3e + 16	bd^4 − 32	
				c^4g + 36	c^4f + 27	
				c^3df − 72	c^3de − 45	
				c^3e^2 − 27	c^2d^3 + 20	
				c^2d^2e + 96		
				cd^4 − 32		

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	$P,\ x^0$
$a^2d^2g^2$	+ 1
$defg$	− 6
df^3	+ 4
e^3g	+ 4
e^2f^2	− 3
$a\,bcdg^2$	− 6
$bcefg$	+ 18
bcf^3	− 12
bd^2fg	+ 12
bde^2g	− 18
be^3f	+ 6
c^3g^2	+ 4
c^2e^2g	− 24
c^2dfg	− 18
c^2ef^2	+ 30
cd^2eg	+ 54
cd^2f^2	− 12
cde^2f	− 42
ce^4	+ 12
d^4g	− 20
d^3ef	+ 24
d^2e^3	− 8
$a^0b^3dg^2$	+ 4
b^3efg	− 12
b^3f^3	+ 8
$b^2c^2g^2$	− 3
b^2ce^2g	+ 30
b^2cef^2	− 24
b^2d^2eg	− 12
$b^2d^2f^2$	− 24
b^2de^2f	+ 60
b^2e^4	− 27
bc^3fg	+ 6
bc^2deg	− 42
bc^2df^2	+ 60
bc^2e^2f	− 30
bcd^3g	+ 24
bcd^2ef	− 84
$bcde^3$	+ 66
bd^4f	+ 24
bd^3e^2	− 24
c^4eg	+ 12
c^4f^2	− 27
c^3d^2g	− 8
c^3def	+ 66
c^3e^3	− 8
c^2d^3f	− 24
$c^2d^2e^2$	− 39
cd^4e	+ 36
d^6	− 8

	$Q,\ x^6$	$R,\ x^6$
a^3dg^2		− 1
efg		+ 9
f^3		− 8
a^2bcg^2		+ 3
$bdfg$		− 24
be^2g		− 45
bef^2		+ 66
c^2fg	− 2	+ 3
$cdeg$	+ 5	+ 48
cdf^2	+ 6	− 12
ce^2f	− 7	− 51
d^3g	− 3	− 16
d^2ef	− 3	+ 36
de^3	+ 4	− 8
$a\,b^3g^2$	0	− 2
b^2cfg	+ 4	+ 12
b^2deg	− 5	+ 192
b^2df^2	− 6	− 48
b^2e^2f	+ 7	− 144
bc^2eg	− 5	− 159
bc^2f^2	− 6	+ 18
bcd^2g	+ 7	− 48
$bcdef$	− 16	+ 24
bce^3	+ 23	+ 279
bd^3f	+ 30	− 48
bd^2e^2	− 33	− 84
c^3dg	− 1	+ 42
c^3ef	+ 36	+ 153
c^2d^2f	− 37	− 36
c^2de^2	− 53	− 399
cd^3e	+ 79	+ 312
d^5	− 24	− 64
a^0b^4fg	− 2	0
b^3ceg	+ 5	0
b^3cf^2	+ 6	0
b^3d^2g	+ 2	− 224
b^3def	+ 22	+ 144
b^3e^3	− 27	+ 54
b^2c^2dg	− 8	+ 336
b^2c^2ef	− 39	− 108
b^2cd^2f	− 50	+ 384
b^2cde^2	+ 107	− 684
b^2d^3e	− 22	+ 144
bc^4g	+ 3	− 126
bc^3df	+ 84	− 648
bc^3e^2	− 21	+ 432
bc^2d^2e	− 102	+ 564
bcd^4	+ 44	− 288
c^5f	− 27	+ 270
c^4de	+ 45	− 450
c^3d^3	− 20	+ 200

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Remaining Coefficients of C, E, G.

C

x^3y	
af	$+2$
be	-6
cd	$+4$
x^2y^2	
ag	$+1$
ce	-9
d^2	$+8$
xy^3	
bg	$+2$
cf	-6
de	$+4$
y^4	
cg	$+1$
df	-4
e^2	$+3$

E

xy	
adg	$+1$
aef	-1
bcg	-1
bdf	-8
be^2	$+9$
c^2f	$+9$
cde	-17
d^3	$+8$
y^2	
aeg	$+1$
af^2	-1
bdg	-3
bef	$+3$
c^2g	$+2$
cdf	-1
ce^2	-3
d^2e	$+2$

G

x^7y	
a^2g	$+1$
abf	$+2$
ace	-19
ad^2	$+8$
b^2e	-6
bcd	$+44$
c^3	-30
x^6y^2	
abg	$+7$
acf	-14
ade	-14
b^2f	0
bce	-21
bd^2	$+112$
c^2d	-70
x^5y^3	
acg	$+7$
adf	-28
ae^2	-14
b^2g	$+14$
bcf	-42
bde	$+168$
c^2e	-105
x^4y^4	
adg	0
aef	-35
bcg	$+35$
bdf	0
be^2	$+105$
c^2f	-105

G

x^3y^5	
aeg	-7
af^2	-14
bdg	$+28$
bef	$+42$
c^2g	$+14$
cdf	-168
ce^2	$+105$
x^2y^6	
afg	-7
beg	$+14$
bf^2	0
cdg	$+14$
cef	$+21$
d^2f	-112
de^2	$+70$
xy^7	
ag^2	-1
bfg	-2
ceg	$+19$
cf^2	$+6$
d^2g	-8
def	-44
e^3	$+30$
y^8	
bg^2	-1
cfg	$+5$
deg	-2
df^2	-8
e^2f	$+6$

Note.—In the tables on this page, a has been treated like the other letters; on the preceding pages, the powers of a have been suppressed except in the first of every series of terms containing a common power of a.

The final result is that we have the values of the invariants B, I, P, W, Z and the leading coefficients of the covariants A, C, D, E, F, G, H, J, K, L, M, N, O, Q, R: also the means of calculating the leading coefficients of the remaining covariants S, T, U, V, X, Y.

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