888.

ON A FORM OF QUARTIC SURFACE WITH TWELVE NODES.

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USING throughout capital letters to denote homogeneous quadric functions of the coordinates (x, y, z, w), we have as a form of quartic surface with eight nodes $\Omega = (*\between U, V, W)^2 = 0$; viz. the nodes are here the octad of points, or eight points of intersection of the quadric surfaces $U=0$, $V=0$, $W=0$; the equation can, by a linear transformation on the functions U, V, W (that is, by substituting for the original functions U, V, W linear functions of these variables), be reduced to the form $\Omega = U^2 + V^2 + W^2 = 0$.

Suppose now that the function Ω can in a second manner be expressed in the like form $\Omega = P^2 + Q^2 + R^2$ (where P, Q, R are not linear functions of U, V, W); that is, suppose that we have identically $U^2 + V^2 + W^2 = P^2 + Q^2 + R^2$, this gives $U^2 - P^2 + V^2 - Q^2 + W^2 - R^2 = 0$; or, writing $U+P$, $V+Q$, $W+R = A$, B, C, and $U-P$, $V-Q$, $W-R = F$, G, H, the identity becomes $AF + BG + CH = 0$; and this identity being satisfied, the equation $\Omega = 0$ of the quartic surface may be written in the two forms

$$\Omega = (A+F)^2 + (B+G)^2 + (C+H)^2 = 0,$$

and

$$\Omega = (A-F)^2 + (B-G)^2 + (C-H)^2 = 0;$$

viz. the quartic surface has the nodes which are the intersections of the three quadric surfaces $A+F=0$, $B+G=0$, $C+H=0$, and also the nodes which are the intersections of the three quadric surfaces $A-F=0$, $B-G=0$, $C-H=0$. We may of course also write the equation of the surface in the form

$$\Omega = A^2 + B^2 + C^2 + F^2 + G^2 + H^2 = 0.$$

An easy way of satisfying the identity $AF + BG + CH = 0$ is to assume

$$A,\ B,\ C,\ F,\ G,\ H = ayz,\ bzx,\ cxy,\ fxw,\ gyw,\ hzw,$$

where the constants a, b, c, f, g, h satisfy the condition $af + bg + ch = 0$; this being so, the functions A, B, C, F, G, H, and consequently the functions $A + F$, $B + G$, $C + H$ and $A - F$, $B - G$, $C - H$ each of them vanish for the four points $(y = 0,\ z = 0,\ w = 0)$, $(z = 0,\ x = 0,\ w = 0)$, $(x = 0,\ y = 0,\ w = 0)$, $(x = 0, y = 0, z = 0)$, or say the points $(1, 0, 0, 0)$, $(0, 1, 0, 0)$, $(0, 0, 1, 0)$, $(0, 0, 0, 1)$. It hence appears that the quartic surface

$$\Omega = a^2y^2z^2 + b^2z^2x^2 + c^2x^2y^2 + f^2x^2w^2 + g^2y^2w^2 + h^2z^2w^2 = 0$$

is a quartic surface with twelve nodes; viz. it has as nodes the last-mentioned four points, the remaining four points of intersection of the surfaces

$$ayz + fxw = 0,\quad bzx + gyw = 0,\quad cxy + hzw = 0,$$

and the remaining four points of intersection of the surfaces

$$ayz - fxw = 0,\quad bzx - gyw = 0,\quad cxy - hzw = 0.$$

The above is the analytical theory of one of the two forms of quartic surface with twelve nodes recently established by Dr K. Rohn in a paper in the *Berichte ü. d. Verhandlungen der K. Sächsische Gesellschaft zu Leipzig*, (1884), pp. 52—60.