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NOTE ON THE CALCULUS OF LOGIC.

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It appears to me that the theory of the Syllogism, as given in Boole's paper, "The Calculus of Logic," *Camb. and Dubl. Math. Jour.*, t. III. (1848), pp. 183—198, may be presented in a more concise and compendious form as follows:

We are concerned with complementary classes, X, X'; viz. these together make up the universe (of things under consideration), $X + X' = 1$; viz. X' is the class not-X, and X the class not-X'.

Any kind whatever of simple relation between two classes (if we attend also to the complementary classes) can be expressed as a relation of total exclusion, $XY = 0$, or as a relation of partial (it may be total) inclusion, YX not $= 0$; viz. the relation $XY = 0$ may be read in any of the forms

No X's are X's,
No Y's are X's,
All X's are not-Y's,
All Y's are not-X's,

and the relation XY not $= 0$ in either of the forms

Some X's are Y's,
Some Y's are X's.

I say the above are the *only* kinds of simple relations; it being understood that X' may be substituted for X, or Y' for Y; so that the example $X'Y = 0$ (all Y's are X's) is the same kind of relation as $XY = 0$; and $X'Y$ not $= 0$ (some Y's are not-X's) the same kind of relation as XY not $= 0$.

Now taking X or X' and Z or Z' for the extreme terms, and Y or Y' for the middle term, of a syllogism; the only combinations of premises are

(1) $XY=0, \quad ZY=0.$

(2) $XY=0, \quad ZY$ not $=0$, therefore $X'Z$ not $=0$.

(3) XY not $=0$, ZY not $=0$.

(4) $XY=0, \quad ZY'=0$, therefore $XZ=0$.

(5) $XY=0, \quad ZY'$ not $=0$.

(6) XY not $=0$, ZY' not $=0$.

And of these, there are (as shown by the third column) only two which give rise to a conclusion (or relation between the extreme terms). As regards the negative cases, this is at once seen to be so; thus $XY=0$, $ZY=0$ (no X's are Y's, no Z's are Y's) leads to no conclusion in regard to X, Z. As regards the positive cases, it is also at once seen that the conclusions do follow; but we may obtain the conclusions by symbolical reasoning, thus

(2) $Y = YX + YX', = YX'$;

therefore $ZY = ZYX'$, not $=0$; therefore ZX' not $=0$.

(4) $XZ = XZY + XZY'$, where on the right-hand side each term (the first as containing XY, the second as containing ZX') is $=0$; that is, $XZ=0$; where the logical signification of each step is obvious.