824.

NOTE ON THE FORMULÆ OF TRIGONOMETRY.

[From the *Johns Hopkins University Circulars*, No. 17 (1882), p. 241.]

THE equations $a = c\cos B + b\cos C$, $b = a\cos C + c\cos A$, $c = b\cos A + a\cos B$, which connect together the sides a, b, c and the angles A, B, C of a plane triangle, may be presented in an algebraical rational form, by introducing in place of the angles A, B, C the functions $\cos A + i\sin A$, $\cos B + i\sin B$, $\cos C + i\sin C$, viz. calling these $\frac{x}{w}$, $\frac{y}{w}$, $\frac{z}{w}$ respectively, or, what is the same thing, writing $2\cos A = \frac{x}{w} + \frac{w}{x}$, $2\cos B = \frac{y}{w} + \frac{w}{y}$, $2\cos C = \frac{z}{w} + \frac{w}{z}$, then the foregoing equations may be written

$$\begin{array}{l}(-2yzw\ \ ,\quad y(z^2+w^2),\quad z(y^2+w^2)\,)\!(\,a,\ b,\ c) = 0,\\ (x(z^2+w^2),\quad -2zxw\ \ ,\quad z(x^2+w^2)\,)\!(\quad ,,\quad) = 0,\\ (x(y^2+w^2),\quad y(x^2+w^2),\quad -2xyw\ \ )\!(\quad ,,\quad) = 0,\end{array}$$

that is, as a system of bipartite equations linear in (a, b, c) and cubic in (x, y, z, w) respectively.

Similarly in Spherical Trigonometry, writing as above for the angles, and for the sides writing in like manner $2\cos a = \frac{\alpha}{\delta} + \frac{\delta}{\alpha}$, $2\cos b = \frac{\beta}{\delta} + \frac{\delta}{\beta}$, $2\cos c = \frac{\gamma}{\delta} + \frac{\delta}{\gamma}$, we have a system of bipartite equations separately homogeneous in regard to (x, y, z, w) and $(\alpha, \beta, \gamma, \delta)$ respectively.