821.

ON THE GEOMETRICAL REPRESENTATION OF AN EQUATION BETWEEN TWO VARIABLES.

[From the *Johns Hopkins University Circulars*, No. 15 (1882), p. 210.]

An equation between two variables cannot be represented in a satisfactory manner by a curve, for this serves only to represent the corresponding *real* values of the variables: to represent the imaginary values the natural course is to represent each variable by a point in a plane, viz. the variable $z, = x + iy$, will be represented by a point the coordinates of which are the components x and y of the variable, and similarly the variable $z', = x' + iy'$, by a point the coordinates of which are the components x' and y' of the variable: the equation between the two variables then establishes a correspondence between the two variable points, or say a correspondence between the planes which contain the two points respectively: and it is this correspondence of two planes which is the proper geometrical representation of the equation between the two variables: to exhibit the correspondence we may in either of the planes draw a network of curves at pleasure, and then draw in the other plane the network of corresponding curves. This well-known theory [can be] illustrated for the case $z' = \sqrt{z^4 - 1}$; taking in the infinite half-plane y positive about the origin as centre a system of semicircles, to these correspond in the infinite plane of $x'y'$ a series of lemniscate-shaped curves: and by means of these it is easy to show in the second plane the path corresponding to a given path of the point $z, = x + iy$, in the first half-plane.