121.

NOTE ON A QUESTION IN THE THEORY OF PROBABILITIES.

[From the *Philosophical Magazine*, vol. VI. (1853), p. 259.]

THE following question was suggested to me, either by some of Prof. Boole's memoirs on the subject of probabilities, or in conversation with him, I forget which; it seems to me a good instance of the class of questions to which it belongs.

Given the probability α that a cause A will act, and the probability p that A acting the effect will happen; also the probability β that a cause B will act, and the probability q that B acting the effect will happen; required the total probability of the effect.

As an instance of the precise case contemplated, take the following: say a day is called *windy* if there is at least w of wind, and a day is called *rainy* if there is at least r of rain, and a day is called *stormy* if there is at least W of wind, *or* if there is at least R of rain. The day may therefore be stormy because of there being at least W of wind, or because of there being at least R of rain, or on both accounts; but if there is less than W of wind *and* less than R of rain, the day will not be stormy. Then α is the probability that a day chosen at random will be windy, p the probability that a windy day chosen at random will be stormy, β the probability that a day chosen at random will be rainy, q the probability that a rainy day chosen at random will be stormy. The quantities λ, μ introduced in the solution of the question mean in this particular instance, λ the probability that a windy day chosen at random will be stormy by reason of the quantity of wind, or in other words, that there will be at least W of wind; μ the probability that a rainy day chosen at random will be stormy by reason of the quantity of rain, or in other words, that there will be at least R of rain.

The sense of the terms being clearly understood, the problem presents of course no difficulty. Let λ be the probability that the cause A acting will act efficaciously; μ the probability that the cause B acting will act efficaciously; then

$$p = \lambda + (1 - \lambda)\,\mu\beta,$$
$$q = \mu + (1 - \mu)\,\alpha\lambda,$$

which determine λ, μ; and the total probability ρ of the effect is given by

$$\rho = \lambda\alpha + \mu\beta - \lambda\mu\alpha\beta;$$

suppose, for instance, $\alpha = 1$, then

$$p = \lambda + (1 - \lambda)\,\mu\beta, \quad q = \mu + \lambda - \lambda\mu, \quad \rho = \lambda + \mu\beta - \lambda\mu\beta,$$

that is, $\rho = p$, for p is in this case the probability that (acting a cause which is certain to act) the effect will happen, or what is the same thing, p is the probability that the effect will happen.

Machynlleth, August 16, 1853.