SPRAWOZDANIA ARCHEOLOGICZNE 65, 2013 PL ISSN 0081-3834

Aleksandr Diachenko*

THE FORMATION OF HIERARCHY: EXPLANATION OF THE PRIMATE RANK-SIZE SETTLEMENTS DISTRIBUTION IN PREHISTORY

ABSTRACT

Diachenko A. 2013. The Formation of Hierarchy: Explanation of the Primate Rank-Size Settlements Distribution in Prehistory. *Sprawozdania Archeologiczne* 65, 67-77.

This paper deals with the formation of the hierarchic settlement systems. The proposed model shows the links between rank-size distribution empirical law and the Xtent model of C. Renfrew and E. Level. The rise of the settlement hierarchies characterized by the primate rank-size distribution could be explained with the increased importance of a single variable, reflecting the political influence of the new centers. Population values increase at least ten times compared to the number of settlements' inhabitants before the formation of the spatial hierarchy. This model could be applied to corrections of population estimates in archaeology.

Key words: hierarchy, population estimates, rank-size rule, settlement systems Received: 9.11.2012; Revised: 3.03.2013; Accepted: 1.08.2013

Numerous real-world phenomena follow power laws that are the functional relationships between two quantities. The rank-size distribution of settlements is one of the best known examples of power-law relations. This empirical law, elaborated by B. Berry (1961; 1967) and propagated by R.J. Chorley, P. Haggett (1967) and other scholars in Analytical Geography, was introduced to Archaeology in the 1970s (Hodder and Orton 1976; Clarke 1977 *et al.*). This study focuses on mathematical relationship between population size and the political status of a settlement over the transition period from egalitarian to ranked

* Institute of Archaeology, Ukrainian National Academy of Sciences, Gieroiv Stalingradu 12, Kyiv-210, Ukraine; oleksandr.diachenko@gmail.com

society. Variables that drive the population growth, forming the primate rank-size distribution of settlements, are examined.

In the beginning of 20th century, German geographer F. Auerbach proposed empirical observations describing the distribution of population in city hierarchies within the integral economic regions. This empirical law known as rank-size rule or Auerbach's rule shows a strong correlation between the population of a city and its rank in the spatial hierarchy. By the middle 20th century rank-size rule was approbated and popularized by K. Zipf (Zipf 1965).

According to the rank-size rule (Zipf's law), the population number of a city at the rank $n(P_n)$ may be found using the following equation:

 $P_n = P_{max} n^{-1}$, (1)

where P_{max} is the population of the largest city in a region.

The rank-size rule was applied to numerous settlement systems, both existing and extinct, in the 1960s — 1970s. The number of these studies somewhat decreased in the 1980s — the beginning of the 1990s. However, interest in the rank-size city distributions is rising again among geographers and archaeologists, including scholars working on non-linear dynamics and power-law functions (Brown *et al.* 2002; 2005). This could be exemplified with S. Brakman and G. Garretsen's paper "The return of Zipf: towards a further understanding of a rank-size distribution", published in Journal of Regional Science (Brakman and Garretsen 1999). The "renaissance" of this idea was probably stimulated by the work of physicians and mathematicians on the problems of 1/f effect, multifractals and self-organized criticality that found similar behavior of the complex systems in physics, biology, geology and other fields (Bak *et al.* 1987; 1988; Bak 1996; Mandelbrot 1999). It should be noted that such analogies were pointed out earlier (Nordbeck 1965; 1971; Woldenberg and Berry 1967; Woldenberg 1969; 1973). The power-law distribution of population groups in huntersgatherer societies was recently shown on wide empirical data (Hamilton *et al.* 2007).

The "classic" rank-size distribution could be graphed as a line sloped to the abscissa axis in a log-log frame of axes (Fig. 1: b). Before being distributed this way, settlements pass statistically significant deviations from the "classic" rank-size distribution. The evolution of the deviations was analyzed in a model proposed by B.J.L. Berry (Berry 1961). The primate rank-size settlement distribution was first indicated by M. Jefferson in 1939. This term characterizes population distribution in the settlement system where the largest settlement dominates by its size (Fig. 1: c).

C.L. Crumley proposed a model describing settlement systems of the early states. Such systems contain a center or city, some functional centers and the served rural settlements. Population values should follow the primate rank-size distribution (Crumley 1976). I. Hodder explained concave rank-size curves as a result of more structured and constrained



Fig. 1. Rank-size settlement distributions: a. Australian type of the settlements distribution; b. "Classical" rank-size distribution; c. Primate rank-size settlements distribution

processes (Hodder 1979). At the present time primate rank-size distribution is typical for countries with lower than average size, simple economic and political structure and for countries with so called dual economy. This type of distribution also characterizes former colonies and rising states (Berry 1961; Haggett 1979, 354–360). R.D. Drennan and C.E. Peterson introduced a mathematical coefficient for describing the shape of a rank-size curve. This coefficient helps to define distributions of different types more precisely (Drenann and Peterson 2004).

The aim of this work is the analysis of the mechanics of primate rank-size distribution formation in early complex societies. We will propose a model describing the demographic changes and then verify it using empirical data.

Assumptions. Assume settlements *I* and *J* had equal populations at the time t_0 . These settlements were "politically" independent from one another at the time t_0 and the areas under political control of *I* and *J* did not extend beyond their resource zones. Settlement *J* became politically dependent from settlement *I* at the time t_1 . The population of settlement *I* increased between t_0 and t_1 . Let P_0 be the population of settlement *I* by the time t_0 and P_I be the population of settlement *I* by the time t_1 . Let P_1 be the population of settlement *J* by the time t_0 and P_I be the population of settlement *I* by the time t_1 .

Assume settlement J was the second largest in the region after settlement I. This assumption allows all the other sites to be ignored. Also it brings our model to a correlation of the populations of settlements I and J or a correlation of the population of settlement I before and after it became a regional center.

Assume that distance between settlements I and J did not change much between t_0 and t_1 . In other words: $D_{IJ} \approx const$.

The model. The political domination between the assumed settlements may be described with the Xtent model proposed by C. Renfrew and E. Level (this model was introduced for the prediction of polities from the centers as an alternative to the application of the rank-size rule):

$$\sqrt{P_I} - \sqrt{P_J} = c D_{IJ},\tag{2}$$

where *c* is the scaling factor (Renfrew and Level 1979).

The "political situation" at the time t_0 may be described with a model synthesized from the Xtent and the gravity model:

$$\sqrt{P_0} + \sqrt{P_1} = c_0 D_{IJ},\tag{3}$$

where c_0 is the scaling factor that does not let political influence spread out of the resource zone (Diachenko, in preparation).

Equations 2 and 3 may be integrated into a single equation system:

$$\begin{cases} c < \frac{\sqrt{P_{I}} - \sqrt{P_{J}}}{D_{IJ}} \\ c_{0} = \frac{(\sqrt{P_{0}} + \sqrt{P_{1}})}{D_{IJ}} \end{cases}$$

$$\tag{4}$$

This means:

$$\frac{c}{c_0} < \frac{\sqrt{P_I} - \sqrt{P_J}}{\sqrt{P_1} + \sqrt{P_0}} \quad \text{or} \tag{5}$$

$$P_I > (\sqrt{P_J} + \frac{c}{c_0} (\sqrt{P_1} + \sqrt{P_0}))^2.$$
(6)

Therefore, settlements I and J are "politically" independent from one another if:

$$P_{I} \le (\sqrt{P_{J}} + \frac{c}{c_{0}} (\sqrt{P_{1}} + \sqrt{P_{0}}))^{2}$$
⁽⁷⁾

The following section examines changes in the population values of the rising centers. Assume that the population of settlement *J* did not changed much from t_0 and t_1 ($P_0 \approx P_1 \approx P_J$). This allows the identification of the population growth coefficient (*M*) by writing the equation 6 as follows:

$$P_I > MP_0,$$
 (8)
where $M = 1 + 4 \frac{c}{c_0} + 4 \frac{c^2}{c_0^2}.$

Thus, the population growth coefficient mainly depends on the values of a scaling factor c. Since the former variable rules the spatial relations between the settlements (Renfrew and Level 1979; Diachenko, in preparation), it reflects the total volume of the inhabited areas' functions. Assuming that the volume of a settlements functions did not increased rapidly at the time of spatial hierarchy formation, and that new centers provided only social services to the villages served, $c/c_0 \rightarrow 1$. Therefore, the populations of the rising regional centers increase at least ten times compared to the population number before the formation of hierarchy:

$$P_l > 10P_0. \tag{9}$$

According to the basic assumptions both abstract settlements should exist for the relatively long time period that includes shifting the social system from egalitarian to ranked society. Therefore, the model proposed above could be applied exclusively to the settlement systems of agriculturalists with stable sedentism.

Analysis of the spatial hierarchy is the immanent characteristic of the Xtent model (Renfrew and Level 1979, 145–147) and the proposed model (equations 7 and 8). One of the variables in the Xtent model reflects the spatial distribution of settlements. The proposed model deals with the demographic changes dynamics. The first model allows any type of the rank-size settlement distribution. Since the ratio of the populations of the two largest settlements in a latter model is more than or equal to 10 ($\ln 10 \approx 2.3$), it exclusively allows the primate rank-size distribution.

As mentioned above, Zipf's law is used to describe the population distribution within the integral regions. Therefore, our model could be applied only to regional analysis. The formation of the more complex settlement systems as the result of the integration of regional inhabited areas is characterized with the transition from the Australian type of distribution to the "classical" rank-size distribution (the term "Australian type" was proposed by P. Haggett for the regions with two or more dominate cities with approximately equal populations) (Fig. 1: a). It could be exemplified with the detailed studies of early states formation in Middle Ages Scandinavia (Thurston 1997; 2001; 2007 *et al.*). It should be noted, that the Xtent model reproduces political control as a factor of the population distribution in the case of the integration of regions as well.

Verification. The demographic data from Oaxaca valley, Mexico was used for the verification of the model. The following section also shows the possible application of the model to the corrections of population estimates in archaeology.

The formation of hierarchy in the Oaxaca valley is dated by the San Jose phase (1150-850 BC). Bigmen or individuals with prestige social status in their communities appeared during a previous phase, Tierras Largas, but society was still egalitarian (Marcus and Flannery 1996, 76-110).

The population estimates for the San Jose Mogote settlement are 71-186 persons during the Tierras Largas phase and 791–1976 persons during the San Jose phase. It should be noted, that the calculations of the settlement's area during the early stages of its existence is complicated because of the latter buildings. Therefore the population estimates are somewhat approximate (Kowalewski *et al.* 1989, 61; Flannery and Marcus 2005, 7). San Jose Mogote became the largest settlement in the region from 1400-1150 BC with a population at least three times more than in the second largest village. During 1150-850 BC, San Jose Mogote became a center of the chiefdom that also included 12–14 settlements with populations equal to or less than 100 persons per village (Marcus and Flannery 1996, 78, 106-108; 2005, 11-12).

Both lower and higher intervals of the population estimates indicate San Jose Mogote's growth over 10 times. However, let us verify the correlation of the values according to the equations 5–7:

$$\frac{c}{c_0} < \frac{\sqrt{791} - \sqrt{100}}{\sqrt{71 + \sqrt{24}}} < 1.36 \tag{10.1}$$

$$\frac{c}{c_0} < \frac{\sqrt{1976} - \sqrt{100}}{\sqrt{182 + \sqrt{62}}} < 1.6 \tag{10.2}$$

$$\frac{c}{c_0} < \frac{\sqrt{1000 - \sqrt{100}}}{\sqrt{170 + \sqrt{57}}} < 1.05 \tag{10.3}$$

J. Marcus and K.V. Flannery proposed the other two population values of San Jose Mogote: near 170-186 people for the Tierras Largas phase and near 1000 people for the San Jose phase (Marcus and Flannery 1996, 78-79, 106; 2005, 7-12). These values correlate as follows:

The Formation of H	lierarchy
--------------------	-----------

$$\frac{c}{c_0} < \frac{\sqrt{1000} - \sqrt{100}}{\sqrt{170} + \sqrt{57}} < 1.05 \tag{10.4}$$

$$\frac{c}{c_0} < \frac{\sqrt{1000 - \sqrt{100}}}{\sqrt{186 + \sqrt{62}}} < 1 \tag{10.5}$$

Since the values of the variable *c* cannot be lower than the values of c_0 , their correlation cannot be lower than 1. Therefore in the last case at least one of the intervals deviates from the model (formula 10.5). Maximal, 186 and 1976 people, and minimal, 71 and 791 people, values for the population of San Jose Mogote correlated well with the model. The values of 186 and 1000 persons are also in agreement with our model. In the first case the settlement's volume of functions increased 1.36-1.6 times, in the latter case it increased 1.05 times.

Let us analyze the values of the scaling factor for the different phases of the settlement's existence. According to the K.V. Flannery's calculations, alluvial soils within the 2.5 km radius from the settlements provided each of the sites with 400 metric tons of maize (Flannery 2009a; 2009b). Assume that 5.0 km was a sum of the two settlements resource zones radiuses. This allows the calculation of the values of variable c_0 . The values we received are as follows:

0.00266 (for a population of 71 persons);

0.00410 (for a population of 170 persons);

0.00430 (for a population of 186 persons).

Now one may calculate the values of the scaling factor *c* for the turning of the settlement into a regional center. According to the formulas 10.1–10.3, the values of the variable cannot exceed:

0.00362 (for a population of 791 persons);

0.00431 (for a population of 1000 persons);

0.00688 (for a population of 1976 persons).

Statistical examination of the population estimates for the interval of 170–186 persons for the population of San Jose Mogote allowed J. Marcus and K.V. Flannery (1996, 78-79) to propose the interval of 170–186 persons for a Tierras Largas phase. Taking into account these values we may limit the maximum population estimates by the interval of 1000– 1976 persons for the San Jose phase.

Conclusion and discussion. Thus, we analyzed the variables that drive population growth, forming the primate rank-size distribution of settlements. The Xtent model of C. Renfrew and E. Level (1979) was applied as a simulation of the "political" relationship between two abstract settlements. In the first case, populations of the abstract villages

73

Aleksandr Diachenko

were assumed to be egalitarian and "politically independent" one from another. In second case, they were modeled as a ranked society, living in two settlements, the center and the satellite. The integration of the related equation into a single equation system led to the development of formulas that describe the relationship between population size and political status of a settlement. These formulas were tested using data from Oaxaca valley.

The fact that the settlement hierarchy indicates the formation of social inequality and the "political" dependence of one settlement on the other is, in general, agreed upon among the specialists in early complex societies. However, the mechanics of the center's formation is a debated problem. M.J. O'Brien, R.L. Lyman and M.B. Schiffer exemplified different approaches to solving the problem with the discussion on the formation of hierarchy in Oaxaca valley in American archaeology. Researchers from the "Penn State group" viewed the formation of hierarchy exclusively as the relation between the human and environment. Taking into account the important role of the environment and technological progress, scholars from "Michigan University group" underlined the key role of the political factors in the centers growth. J. Marcus and K.V. Flannery even published a photograph of the well-known monastery in Greece, located on a 400m high rock, asking B. Sanders from the "Penn State group" to show the fertile soil ("the key factor of the settlement's location") on this picture (O'Brien *et al.* 2005, 200–205).

The results of our model correlate with the views of J. Marcus and K.V. Flannery together with the viewpoint on the formation of hierarchy in Marxist archaeology. This correlation was made possible with the development of V. Gordon Childe's ideas in Marxist archaeology and, partly, in Processual archaeology (see more: Biehl *et al.* 2002; Renfrew and Bahn 2004). According to equations 7 and 8, the population growth is determined with the increase of values of the single variable — the scaling factor *c* or volume of the political influence expressed mathematically. More simply, the variable reflects the total volume of the economical, administrative, ideological and military functions of the emerging centers. In the case of the formation of hierarchy in early complex societies the volume of a settlement's functions did not increase rapidly, and, most probably, the new centers mostly shared the social functions.

Correlation between the scaling factor from the Xtent model and the population values may be explained using the Lowry's model that shows the influence of the activities of one kind to the activities of the other kind. In the economy of cities the cascade effect of the Lowry's model appears in the limitation of employment in the primary sector accompanied with the increase of jobs in the secondary sector together with the growth of tertiary and quaternary sectors (Haggett 1979, 326–327). In the case of early cities this effect could be reached only with the division of labor and the concentration of surplus.

W. Christaller's Central Place Theory also relates population to the number of a city's central functions (Christaller 1966). According to the Central Place Theory, the distribution of population is characterized by a Pareto distribution function with an exponent of 1 (Beckmann 1958; Beckmann and McPherson 1970). In some circumstances the distribution of population in central place hierarchies may be characterized with the "classical" rank-size city distribution (Beckmann 1958). This empirical data that supports this idea were published since the late 1950es (Berry and Garrison 1958; Berry 1967, 26–40 and others).

Of course, the rank-size rule, as any "ideal model", somewhat schematizes the population distribution. According to this rule, settlements of the fixed rank should have fixed populations. In reality latter values vary in the result of influence by different factors, including the simplest case — the population reproduction (Woldenberg, personal communication on 18.01.2011). Therefore, arguments for the existence of social ranking based exclusively on the settlement systems seem to be complicated, and such studies usually require the inclusion of extra data (Flannery 1998, 16–21). We hope that the proposed model will bring us one step closer to solving this problem.

Future work on the calculations of the scaling factor values depending on different economic and political systems seems to be a priority for the model development. It might be used for future modeling of the development of settlement systems. Another important problem for future work is a separation of economic and political factors that impact the settlements' growth. The proposed model may be applied to the correction of population estimates in archaeology.

Acknowledgements. This paper was performed within the framework of a Fulbright project "Ancient agriculturalists of the Carpatho-Dnipro region: a general model of the socio-economic adaptation". I am sincerely grateful to Professor Michael J. Woldenberg for the interesting and useful communication and numerous consultations on mathematical modeling in geography. I am grateful to Dr. Dustin M. Keeler for his comments and correction of the English text of this paper. I would also like to thank the anonymous reviewer for his valuable comments and remarks.

References

Bak P. 1996. How Nature Works: the Science of Self-Organized Criticality. New-York: Springer-Verlag.

- Bak P., Tang C. and Wiesenfeld K. 1987. Self-organized criticality: an explanation of 1/f noise. *Physical Review Letters* 59(4), 381–384.
- Bak P., Tang C. and Wiesenfeld K. 1988. Self-organized criticality. *Physical Review A* 38(1), 364–375. Beckmann M.J. 1958. City hierarchies and the distribution of city size. *Economic Development and*
- Culture Change 6(3), 243–248.
- Beckmann M.J. and McPherson J.C. 1970. City size distribution in a central place hierarchy: an alternative approach. *Journal of Regional Science* 10(1), 25–33.
- Berry B.J.L. 1961. City size distribution and economic development 1. *Economic Development and Culture Change* 9(4), 573–588.

- Berry B.J.L. 1967. Geography of Market Centers and Retail Distribution. Englewood Cliffs, New Jersey: Prentice-Hall.
- Berry B.J.L. and Garrison W.L. 1958. A note on Central Place Theory and the range of a good. *Economic geography* 34(4), 304–311.
- Biehl P.F., Gramsch A. and Marcianic A. (eds.). 2002. Archaeologies of Europe / History, Methods and Theories. Series Tübinger Archäologische Taschenbücher. Münster, New York, München, Berlin: Waxmann.
- Brakman S. and Garretsen H. 1999. The Return of Zipf: Towards a Further Understanding a Rank-Size Distribution. *Journal of Regional Science* 39(1), 183–213.
- Brown J.H., Gupta V.K., Li B.-L., Milne B.T., Restrepo C. and West G.B. 2002. The fractal nature of nature: power laws, ecological complexity and biodiversity. *Philosophical Transactions of the Royal Society B* 357(1421), 619–626.
- Brown C.T., Whitschey W.R.T. and Leibovitch L.S. 2005. Broken past: fractals in archaeology. *Journal of Archaeological Method and Theory* 12(1), 37–78.
- Christaller W. 1966. *Central Places in Southern Germany*. Englewood Cliffs, New Jersey: Prentice-Hall.
- Chorley R.J. and Haggett P. (eds.). 1967. Models in Geography. London: Methuen.
- Clarke D.L.L. (Ed.). 1977. Spatial Archaeology. Cambridge: Peterhouse.
- Crumley C.L. 1976. Toward a locational definition of state systems of settlement. *American Anthropologist. New series* 78(1), 59–73.
- Diachenko A. Settlement's growth as a fractal. In preparation.
- Drennan R.D. and Peterson C.E. 2004. Comparing archaeological settlement systems with rank-size graphs: a measure of shape and statistical confidence. *Journal of Archaeological Science* 31(5): 533–549.
- Flannery K.V. 1998. The ground plans of archaic states. In Feinman G.M. and Marcus J. (eds.). Archaic States. Santa Fe, New Mexico: School of American Research Press, 15–58.
- Flannery K.V. 2009a. Empirical determination of site catchments in Oaxaca and Tehuacán. In Flannery K.V. (ed.). *The Early Mesoamerican Village. Updated Edition*. Walnut Creek, California: Left Coast Press, 103–116.
- Flannery K.V. 2009b. Linear Stream Patterns and Riverside Settlement Rules. In Flannery K.V. (ed.). The Early Mesoamerican Village. Updated Edition. Walnut Creek, California: Left Coast Press, 173–189.
- Flannery K.V. and Marcus J. 2005. Excavations at San José Mogote 1. The Household Archaeology. Memoirs of the Museum of Anthropology, University of Michigan 13(40). Michigan: Ann Arbor.
- Haggett P. 1979. *Geography. A Modern Synthesis*. New York, Hagerstown-Philadelphia, San Francisco, London: Harper and Row.
- Hamilton, M. J., Milne, B. T., Walker, R. S., Burger O., & Brown, J. H. 2007. The complex structure of hunter-gatherer social newtwork. *Proceedings of the Royal Society B* 274(1622), 2195– 2202.

Hodder I. 1979. Simulating the growth of hierarchies. In Renfrew C. and Cooke K.L. (eds.), Transformations. Mathematical Approach to Cultural Change. New York, San Francisco, London: Academic Press, 117–144.

Hodder I. and Orton C. 1976. Spatial Analysis in Archaeology. Cambridge: Cambridge University Press.

 Kowalewski S.A., Feinman G.M., Finsten L., Blanton R.E. and Nicholas L.M. 1989. Monte Albán's Hinterland 2. Prehispanic Settlement Patterns in Tlacolula, Etla, and Ocotlan, the Valley of Oaxaca, Mexico. *Memoirs of the Museum of Anthropology, University of Michigan* 1(23). Michigan: Ann Arbor.

- Mandelbrot B.B. (1999). *Multifractals and 1/f Noise: Wild Self-Affinity in Physics (1963–1976)*. New-York: Springer-Verlag.
- Marcus J. and Flannery K.V. 1996. Zapotec Civilization. How urban Society Evolved in Mexico's Oaxaca Valley. London: Thames and Hudson.
- Nordbeck S. 1965. The law of allometric growth. In Nystuen J.D. (ed.). *Michigan Inter-University Community of Mathematical Geographers* 7. Ann Arbor: Department of Geography, University of Michigan.
- Nordbeck S. 1971. Urban allometric growth. Geografiska Annaler B 53(1), 54-67.
- O'Brien M.J., Lyman R.L. and Schiffer M.B. 2005. *Archaeology as a Process. Processualism and its Progeny.* Salt Lake City: University of Utah Press.
- Renfrew C. and Bahn P.G. 2004. Archaeology: Theories, Methods and Practice. Fourth Edition. New York.
- Renfrew C. and Level E.V. 1979. Exploring dominance: predicting polities from centers. In Renfrew C. and Cooke K.L. (eds.). *Transformations. Mathematical Approach to Cultural Change*. New York, San Francisco, London: Academic Press, 145–167.
- Thurston T.L. 1997. Historians, prehistorians, and the tyranny of historic record: Danish State formation through documents and archaeological data. In Kepecs S. and Kolb M. (eds.). *New Approaches to Combining the Archaeological and Historical Records. Special Guest Edited Volume of the Journal of Archaeological Method and Theory* 4, 239–263.
- Thurston T.L. 2001. Landscapes of Power, Landscapes of Conflict: State Formation in the South Scandinavian Iron Age. New York: Kluwer Academic / Plenum Publishers.
- Thurston T. 2007. The location of power and the art of resistance: spatial analysis and social theory in regional archaeology. In Salisbury R.B. and Keeler D. (eds.). *Space Archaeology's Final Frontier? An Intercontinental Approach*. Newcastle: Cambridge Scholars Publishing, 159–191.
- Woldenberg M.J. 1969. Spatial order in fluvial systems: Horton's laws derived from mixed hexagonal hierarchies of drainage basin areas. *Geological Society of American Bulletin* 80(1), 97–112.
- Woldenberg M.J. 1973. An allometric analysis of urban land use in the United States. *Ekistics* 215, 282–290.
- Woldenberg M.J. and Berry B.J.L. 1967. Rivers and central places: analogous systems? Journal of Regional Science 7(2), 129–139.
- Zipf G.K. 1965. Human Behavior and the Principle of Least Effort. An Introduction to Human Ecology. New York, London: Hafner Publishing Company.