STOCHASTIC ROTORDYNAMICS: DIRECT AND INVERSE PROBLEMS

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Predicting transverse random vibrations of shafts in rotating machinery may be of importance for applications with high environmental dynamic loads on supports, e.g. in transport. Design of a turbopump for liquid-propellant rocket engine may be quoted as an example whereby operation of the shaft close to its instability threshold was of concern because of increased sensitivity of the whole system shaft-machine-vehicle to such loads [1]. On the other hand, small random vibration components may sometimes be observed in stationary fluid rotating machinery (turbines, fans, etc) – see Figure 1 [2]. The measurable random vibration signals (e.g. due to turbulence in working fluid) may then be used with advantage for on-line condition monitoring of the shaft during its steady operation at a given rotation speed. Survey of recent research results [3 – 8] in transverse random vibrations of rotating shafts is presented here with solutions to various direct and inverse random vibration problems for simple single-disk shafts with potential instability due to internal or "rotating" damping (the latter may also provide a simplified representation for destabilizing nonconservative fluid or magnetic forces).

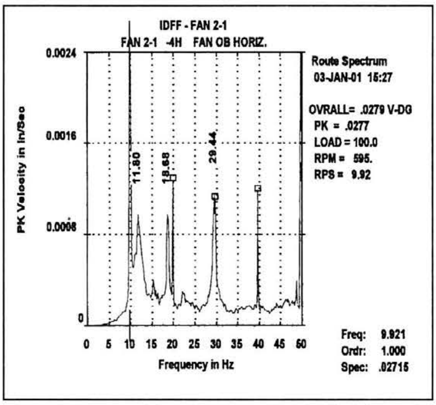

Fig. 1. Spectral density of vibration signal from bearing of a large fan with dominant peak at rotational frequency 9.92 Hz and neighboring peak at the shaft's resonance (critical speed) at 11.80 Hz [2].

Transverse random vibrations of a single-disk two-degrees-of-freeedom rotating shaft with both external ("nonrotating") and internal ("rotating") damping are considered with the latter type of damping being a potential source for dynamic instability. Analytical solutions for mean square transverse displacements $\left\langle X^2\right\rangle, \left\langle Y^2\right\rangle$ are obtained for linear vibrations during operation below instability threshold as well as for tilting oscillations with gyroscopic effect of the disk being represented [3]. The results illustrate: i) magnification of the response intensity with increasing rotation speed ν ; thus "universal" magnification law for the mean square whirl radius $\left\langle R^2\right\rangle = \left\langle X^2\right\rangle + \left\langle Y^2\right\rangle$ is found as $\langle R^2 \rangle = \langle R^2 \rangle_{\nu=0} \cdot (1-\nu/\nu_*)^{-1}$ where ν_* is the instability threshold rotation speed; ii) general trend towards equalizing partition of response energy between two perpendicular directions: even in case of uniaxial excitation ratio of mean square responses in nonexcited and excited directions approaches unity with $\nu/\nu_* \to 1$ that is, with approaching state of forward whirl. Furthermore, coherence functions of responses in two perpendicular directions are calculated as functions of rotation speed [3, 8]. These results are used to develop procedures for on-line evaluation of the shaft's stability margin that rely on response signals measurements during steady operation at any rotation speed below ν_*.

The shaft is also considered for the case where its "rotating" damping is subject to slow temporal random variations that may lead to potential "short-term" instability [6]. The corresponding transient response analysis as based on the Krylov-Bogoliubov averaging and parabolic approximation for peaks of the random damping factor provides probabilistic predictions for outbreaks in the shaft's radius of whirl. Procedure for estimating statistical properties of damping variations from the observed intermittent response with outbreaks, or "puffs" is outlined also.

Lateral vibrations of a single-disk shaft are considered with stiffening nonlinearity taken into account either in restoring force or in damping [4, 5, 7]. Certain exact and approximate analytical solutions for joint probability density function (PDF) of displacements and velocities in two perpendicular directions are obtained. The results may be used to evaluate, from on-line response measurements, whether the shafts operates below or above its instability threshold ν_*. Specifically, PDF *w(V)* of the squared whirl radius $V = X^2 + Y^2$ should be measured for randomly vibrating shaft. If *w(V)*is monotonically decreasing then the shaft is stable in the linear approximation, otherwise its observed response represent self-excited oscillations with superimposed random vibrations.

Finally, first-passage problem is considered for a lightly damped nonlinear shaft. The equations of motion are reduced using stochastic averaging. Then analytical solution for the expected time for crossing giving level by the whirl radius is derived. It can be applied for the important case where stable self-oscillations of the shaft may exist within some range of rotation speeds below instability threshold so that random excitation may lead to a "hard" self-excitation of whirl.

References

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