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DEMONSTRATION OF A NEW FUZZY-SETS METHOD FOR CLASSIFICATION OF VESSELS ON A SAMPLE FROM THE NITRA-LUPKA SITE

Abstract: The aim of the paper is to present a new statistical method which records the shape of vessels and sorts those vessels into groups based on their shape. This new statistical method – the method of fuzzy similarity allows us to find the most similar vessels according to their shape regardless of their size. It is presented on a sample group of vessels from the Nitra-Lupka cemetery. Cemetery and pottery kilns from the $9^{th}/10^{th}$ c. in Nitra-Lupka are located in the foreland of the hillfort-fortification. Vessels revealed in graves were burned in these pottery kilns. Vessels are of high quality, although they were produced only on slow-rotating potter's wheel. The results obtained by using of the new method were compared with the results of standard cluster analysis for four different cases of groups of variables. Chi-square-test was used to compare the two selected methods for the classification of vessels into groups.

Keywords: Slavic pottery, Great Moravian State, Slovakia, classification of shape of vessels, fuzzy cluster analysis, chi-square test.

Abstrakt: Cieľom príspevku je prezentovať novú štatistickú metódu, ktorá zaznamenáva tvar nádob a delí ich do skupín podľa ich tvaru. Táto nova štatistická metóda – metóda fuzzy podobnosti dovoľuje nám nájsť najpodobnejšie nádoby podľa ich tvaru bez ohľadu na ich veľkosť. Táto nová metóda je prezentovaná na vzorke nádob z pohrebiska Nitra-Lupka. Pohrebisko a hrnčiarske pece datované do 9./10. stor. v Nitre-Lupke sú situované v predpolí fortifikácie hradiska. Nádoby odkryté v hroboch boli vypaľované v tunajších hrnčiarskych peciach. Nádoby sú vysokej kvality, hoci boli vyrobené len na pomaly rotujúcom hrnčiarskom kruhu. Výsledky získané použitím novej metódy boli porovnané s výsledkami dosiahnutými za pomoci klasickej clustrovej metódy pre štyri odlišné skupiny premenných. Chi-kvadrát test bol použitý na porovnanie dvoch vybraných metód pre klasifikovanie nádob v rámci skupín.

Kľúčové slová: slovanská keramika, Veľkomoravská ríša, Slovensko, klasifikácia tvarov nádob, fuzzy cluster analýza, chí-kvadrat test.

INTRODUCTION

Ceramic vessels are common in graves in Slavic cemeteries in central Europe, Slovakia included, from the 9^{th} to the 11^{th} c. The phenomenon is connected with pagan ideas of life after death. The dead were usually given one vessel, but sometimes several vessels were found in one grave and often one of these vessels had been broken intentionally during the burial ceremony.

Ceramic vessels have been studied extensively, not only in formal terms, but also with regard to other features, including technical, metrical and qualitative

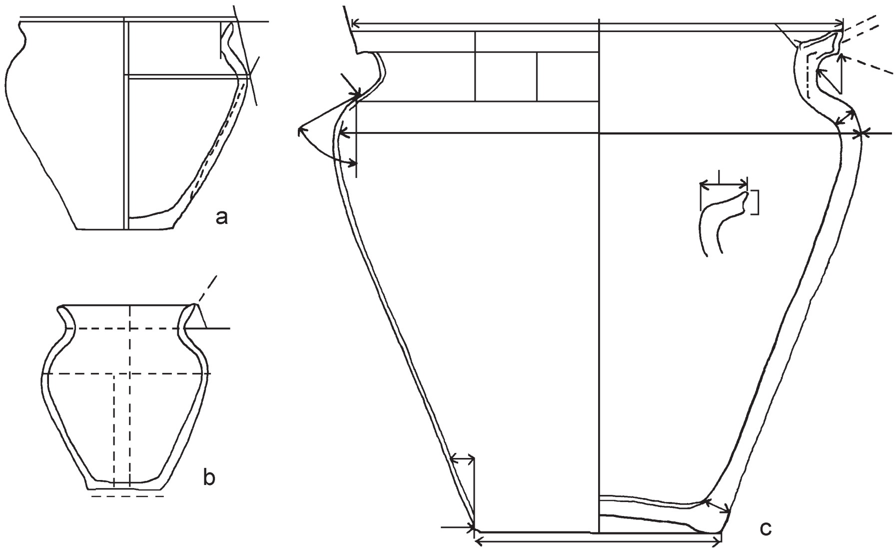

Fig. 1. Choice of diverse systems for quantitative characterisation of vessels.
After U. Dymaczewska 1969, Fig. 1 (a);
M. Parczewski 1977, Fig. 2 (b); A. Buko 1990, Fig. 104c (c)

Obr. 1. Výber rôznych systémov pre kvantitatívnu charakteristiku nádob.
Podľa U. Dymaczewskej 1969, obr. 1 (a);
M. Parczewského 1977, obr. 2 (b); A. Buka 1990, obr. 104c (c)

features (see F. Curta 2008; C. Orton, A. Tyers, A. Vince 1993; A. Gilboa, A. Karasik, L. Sharon, U. Smilansky 2004; A. Karasik, L. Bitton, A. Gilboa, I. Sharon, U. Smilansky 2010; A. Karasik, U. Smilansky 2011; P.M. Rice 1987; A. Sheppard 1956). Slavic pottery has also been studied extensively, especially with regard to metrical properties, e.g. A. Buko 1979; U. Dymaczewska 1969; G. Fusek 1994; 1995; J. Macháček 2001; 2002; U. Maj 1990; M. Parczewski 1977; A. Tirpáková, D. Bialeková, I. Vlkolinská 1989; A. Tirpáková, I. Vlkolinská 1992; M. Trzeciecki 2005; 2009; I. Vlkolinská 1994; 1995; 2007.

Some of the metrical properties of Slavic ceramics are presented below (Fig. 1). The literature reviewed was geared to the needs of the present study of vessels from graves in Slovakia, but of course the incidence of ceramic vessels is not limited to this territory. Many vessels were excavated in graves in Poland, too, for example (M. Trzeciecki 2005).

Nitra was a very important centre in the Great-Moravian period. One of the localities was a hillfort, Nitra-Lupka, which served as a lookout post and refuge. It consisted of a hillfort, which was not excavated, and two groups of pottery kilns (B. Chropovský 1959) and a cemetery (B. Chropovský 1962), these being situated outside the hillfort, but near the fortification. The graves discovered in Nitra-Lupka counted 92 in all, including one in front of the kiln. The body in the latter case had been buried in flexed position, the legs almost in the heating chamber, and there were no furnishings. The collection of pottery vessels from these graves was extensive, altogether 53 vessels, most often two items per burial, but occasionally three in a grave as well. Restorable vessels were discovered in only 38 graves.

An examination of the finds could suggest that the local potters were buried in this cemetery along with their family members. Vessels appeared especially in the female and children graves. There were no weapons in the graves (B. Chropovský 1962, Table 1), ordinary jewellery was present in fair abundance, especially earrings of the Nitra-Lupka-type (B. Chropovský 1962, pp. 211–217); one of the earrings was in the pottery kiln. Most of these jewels were made of bronze and only a small part was made of silver and gold. Nitra-Lupka-type ornaments, especially earrings, were discovered also in other Great-Moravian cemeteries in Slovakia and Moravia, but the main concentration was observed in Nitra-Lupka. Parallels for the earrings come from the northern part of Yugoslavia, as well as from Bulgaria and Romania (T. Štefanovičová 1990, pp. 217, 218; P. Langó 2012), supporting the assumption that the potters from Nitra-Lupka came from there (T. Štefanovičová 1990, p. 219; P. Langó 2012).

Importantly, a comparison of the ceramics from the Nitra-Lupka graves and kilns shows a shared origin. However, while the kilns yielded only vessel fragments (I. Vlkolinská 2002), complete vessels were found in the graves (B. Chropovský 1965; I. Vlkolinská 2007; 2009a; 2009b) and these complete vessels from the cemetery became the subject of our analysis. The goal was to test and compare shapes of vessels with a new mathematical method, the fuzzy cluster method, demonstrated on vessels from the Nitra-Lupka cemetery site.

The vessels in the graves were probably connected with the afterlife beliefs of the inhabitants. They contained symbolic food or liquids for life after death. The main form of the vessels from the graves in Nitra-Lupka can be described as a high and wide pot with a wide neck. The most usual decoration consisted of a wave on the neck and circumferential grooves below it (Fig. 2a). These were the vessels that were found mostly in the backfill of the grave, presumably broken intentionally during the funerary ceremonies. Their height was from 150 mm to 200 mm and occasionally more than 200 mm. The smaller slender vessels, constituting 34% of the ceramic finds from this cemetery (Fig. 2b), were placed with the bodies of the deceased in the graves. They were smaller, from 50 mm to 150 mm high, and in some cases decorated in the same way as the intentionally broken ones. It is noteworthy that all the decoration was executed with a single-spike tool and not the comb-like one. About 28% of the vessels had decoration consisting of grooves and garlands only. Very seldom decoration was recorded on the inside rim of the vessels.

The vessels were very regular and were produced on a slow-rotating potter's wheel, as evidenced by numerous plastic marks on the bottoms of about 40% of the vessels (I. Vlkolinská 2007; 2009b). The marks were mostly basic geometric shapes or simple combinations of these. No two identical marks were found on pots from the cemetery in Nitra-Lupka and more importantly, there were no differences between the pots with and without marks. One should conclude that they were produced by the same pottery manufactory (I. Vlkolinská 2012).

For the sake of comparison, the kilns yielded mostly fragmented pots and bowls, for the most part not to be reconstructed as complete vessels, often bearing

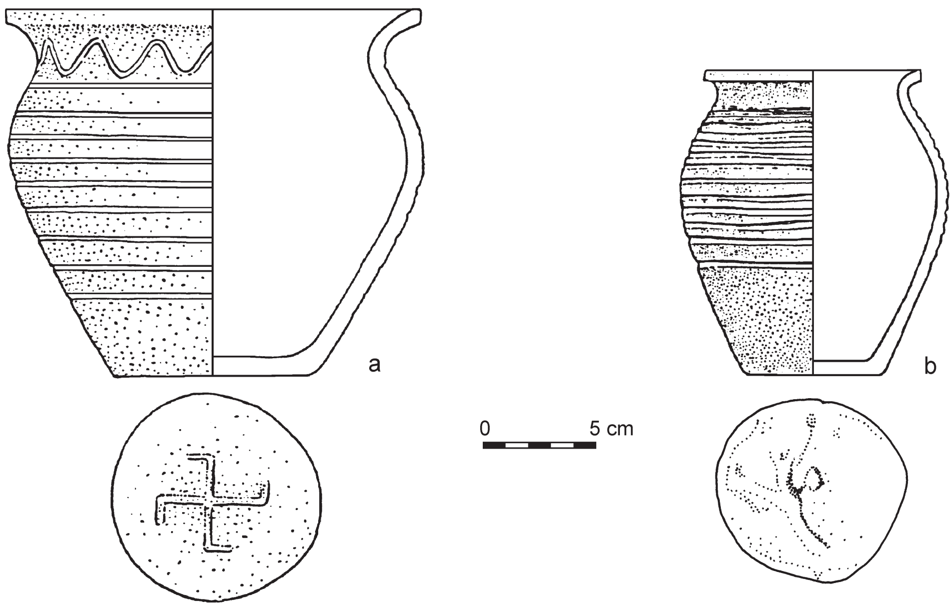

Fig. 2. Choice of main types in Nitra-Lupka.
Drawing M. Pekáriková

Obr. 2. Výber hlavných typov na pohrebisku Nitra-Lupka.
Rys. M. Pekáriková

decoration similar to that found on vessels from the graves, but also quite frequently made with the comb-like tool (I. Vlkolinská 2002; 2005; 2009a). Decorative motifs of wavy lines and horizontal lines or grooves were similar to those found on vessels from the graves.

The present contribution demonstrates a new statistical method for classification of vessels based on their shape. This method of fuzzy similarity allows us to find the most similar vessels according to their shape regardless of their size. The method is presented on a sample group of vessels from the Nitra-Lupka cemetery as described above (Figs 3–10; I. Vlkolinská 2007).

MATERIALS AND METHODS

The objective was to develop an algorithm for sorting vessels according to their shape, using a new method for finding vessel form similarities. Having two distinct groups of vessels distinguished on the grounds of a visual comparison of shapes, we searched for confirmation for our hypothesis.

The new algorithm for vessel classification according to shape is based on a modification of the fuzzy method (described by F.H. Cheng, W.H. Hsu, C.A. Chen 1989) used to recognize handwritten Chinese characters. This is a universal technique for describing shape that does not take size to be significant. The fact that the method

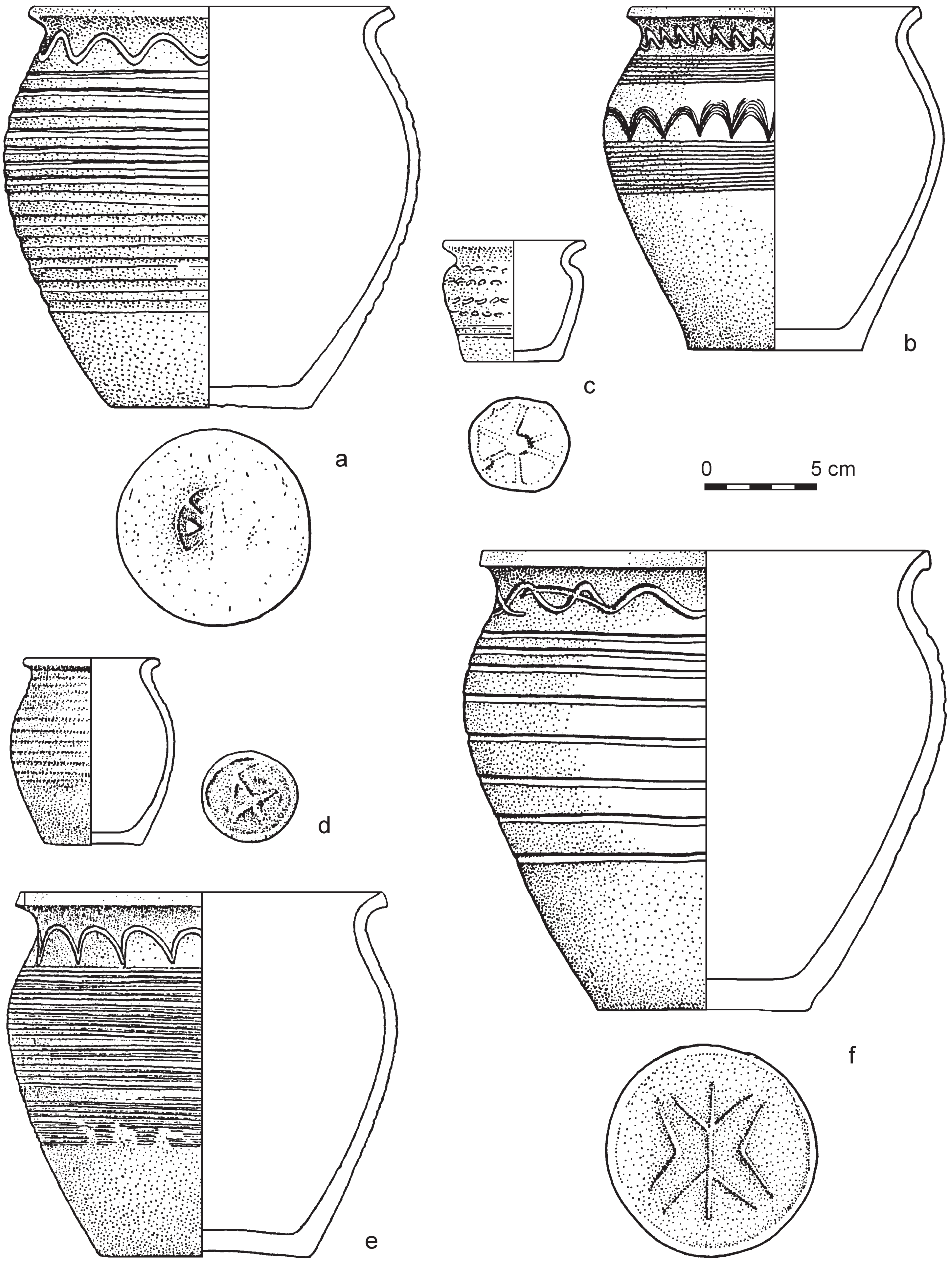

Fig. 3. Vessels from graves
a – grave 7; b, e – grave 11; c, d – grave 10; f – grave 15.

Drawing M. Pekáriková

Obr. 3. Nádoby z hrobov
a – hrob 7; b, e – hrob 11; c, d – hrob 10; f – hrob 15.

Rys. M. Pekáriková

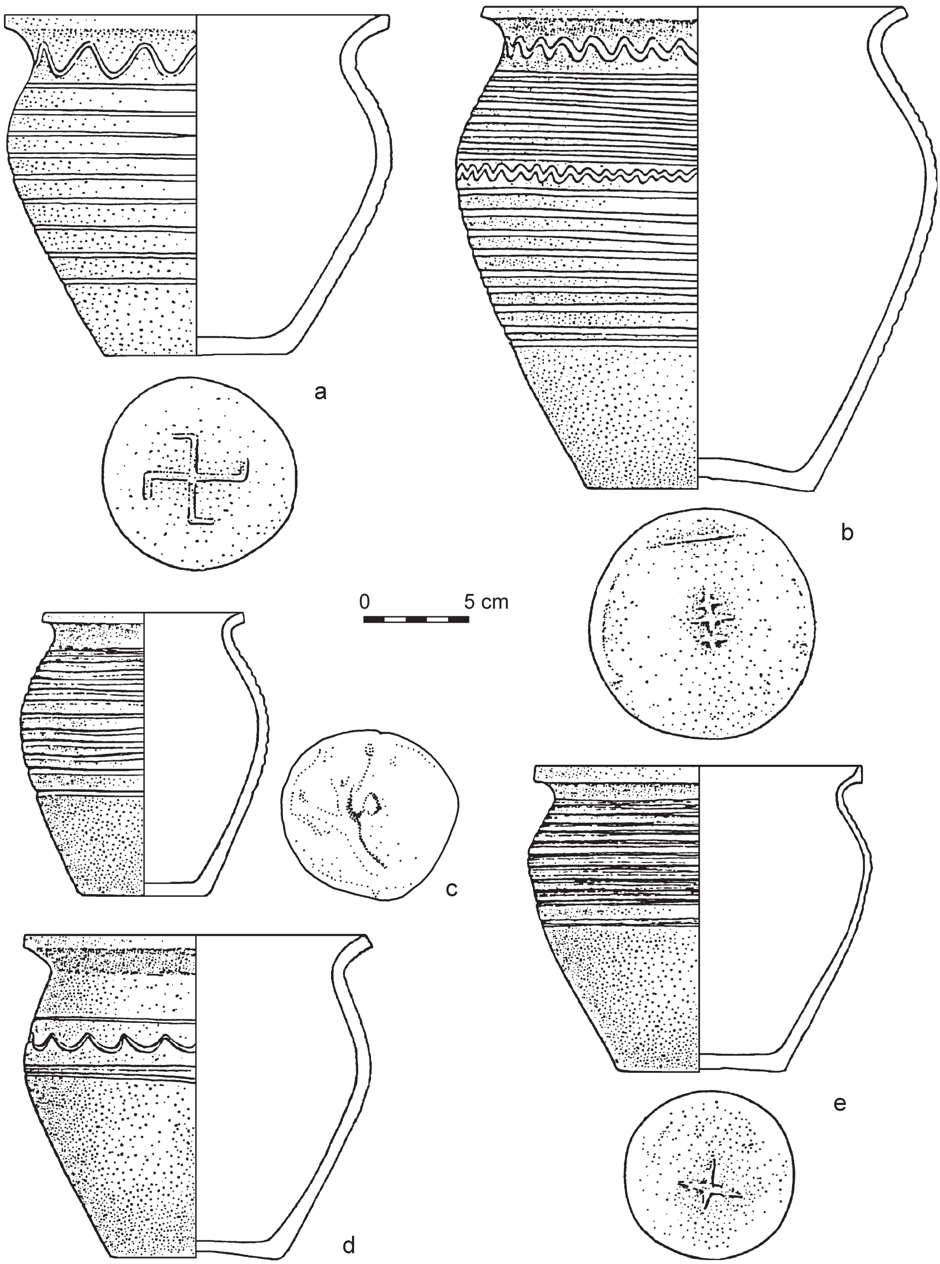

Fig. 4. Vessels from graves
a – grave 19; b – grave 17; c, e – grave 24; d – grave 21.

Drawing M. Pekáriková

Obr. 4. Nádoby z hrobov
a – hrob 19; b – hrob 17; c, e – hrob 24; d – hrob 21.

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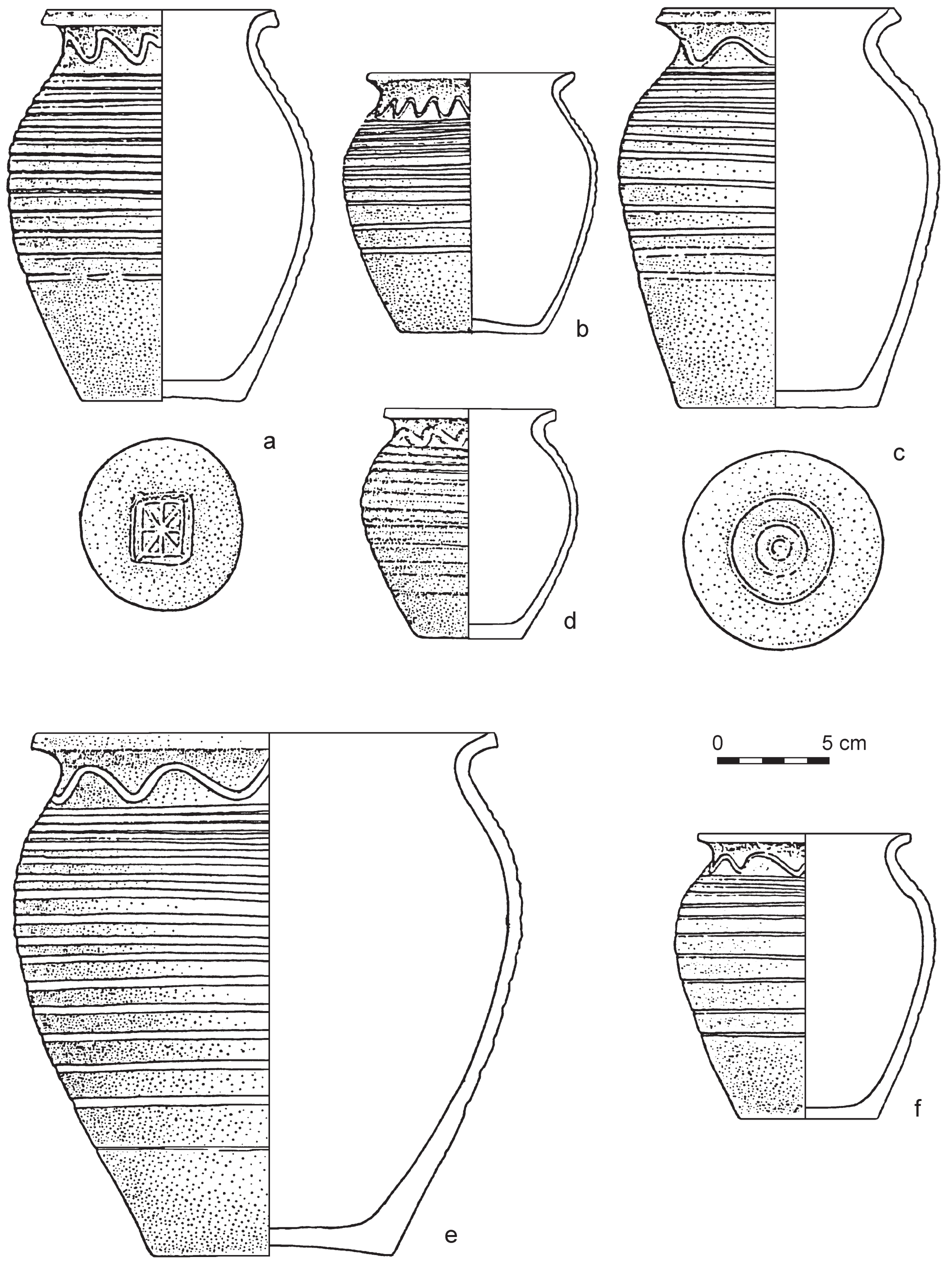

Fig. 5. Vessels from graves
a, b – grave 30; c, f – grave 43; d – grave 51; e – grave 42.

Drawing M. Pekáriková

Obr. 5. Nádoby z hrobov
a, b – hrob 30; c, f – hrob 43; d – hrob 51; e – hrob 42.

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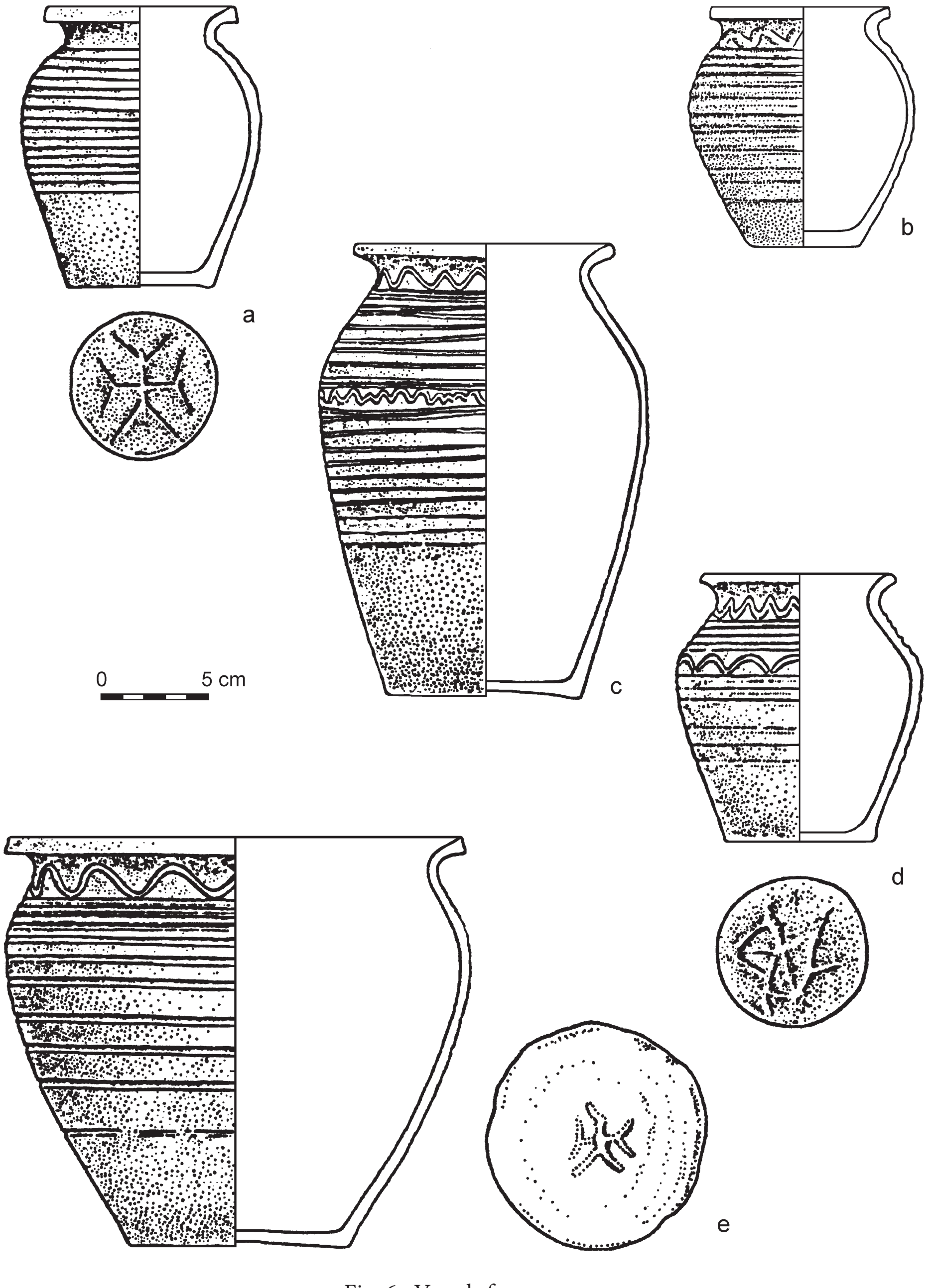

Fig. 6. Vessels from graves
a, e – grave 46; b-d – grave 51.

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Obr. 6. Nádoby z hrobov
a, e – hrob 46; b-d – hrob 51.

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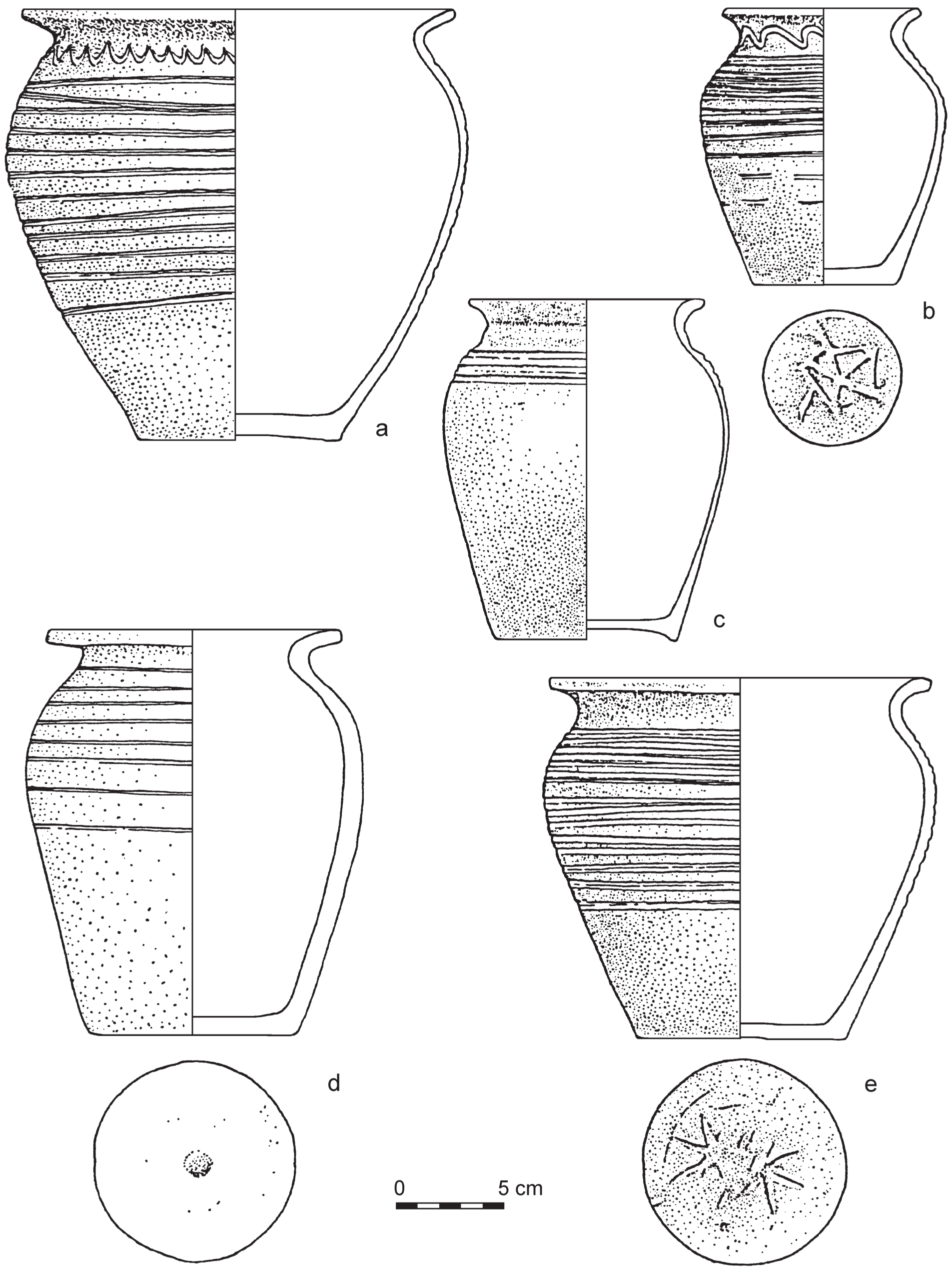

Fig. 7. Vessels from graves
a – grave 64; b – grave 71; c – grave 80; d, e – grave 75.

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Obr. 7. Nádoby z hrobov
a – hrob 64; b – hrob 71; c – hrob 80; d, e – hrob 75.

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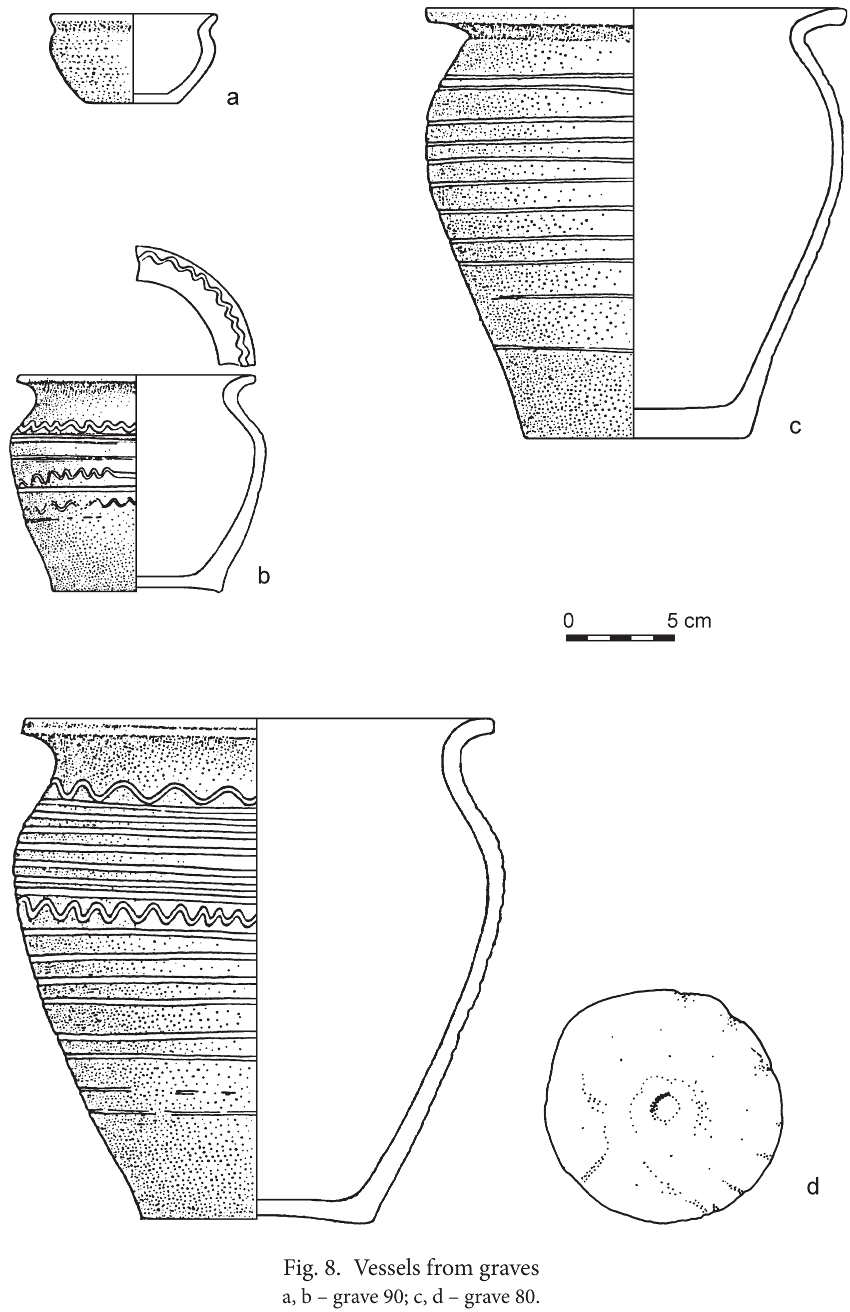

Fig. 8. Vessels from graves
a, b – grave 90; c, d – grave 80.

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Obr. 8. Nádoby z hrobov
a, b – hrob 90; c, d – hrob 80.

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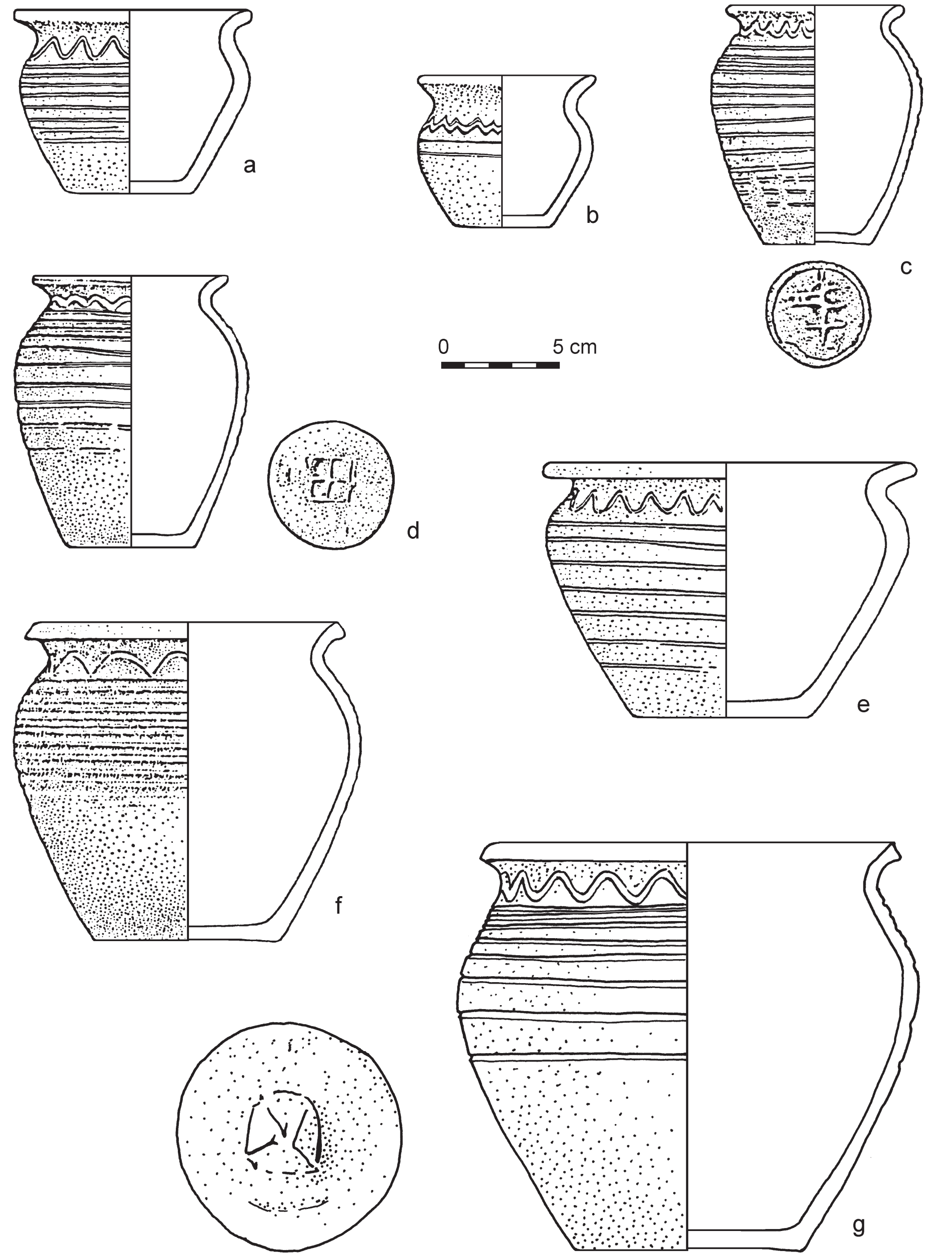

Fig. 9. Vessels from graves
a – grave 18; b – grave 29; c – grave 31; d – grave 40; e – grave 53; f – grave 57; g – grave 16.
Drawing M. Pekáriková

Obr. 9. Nádoby z hrobov
a – hrob 18; b – hrob 29; c – hrob 31; d – hrob 40; e – hrob 53; f – hrob 57; g – hrob 16.
Rys. M. Pekáriková

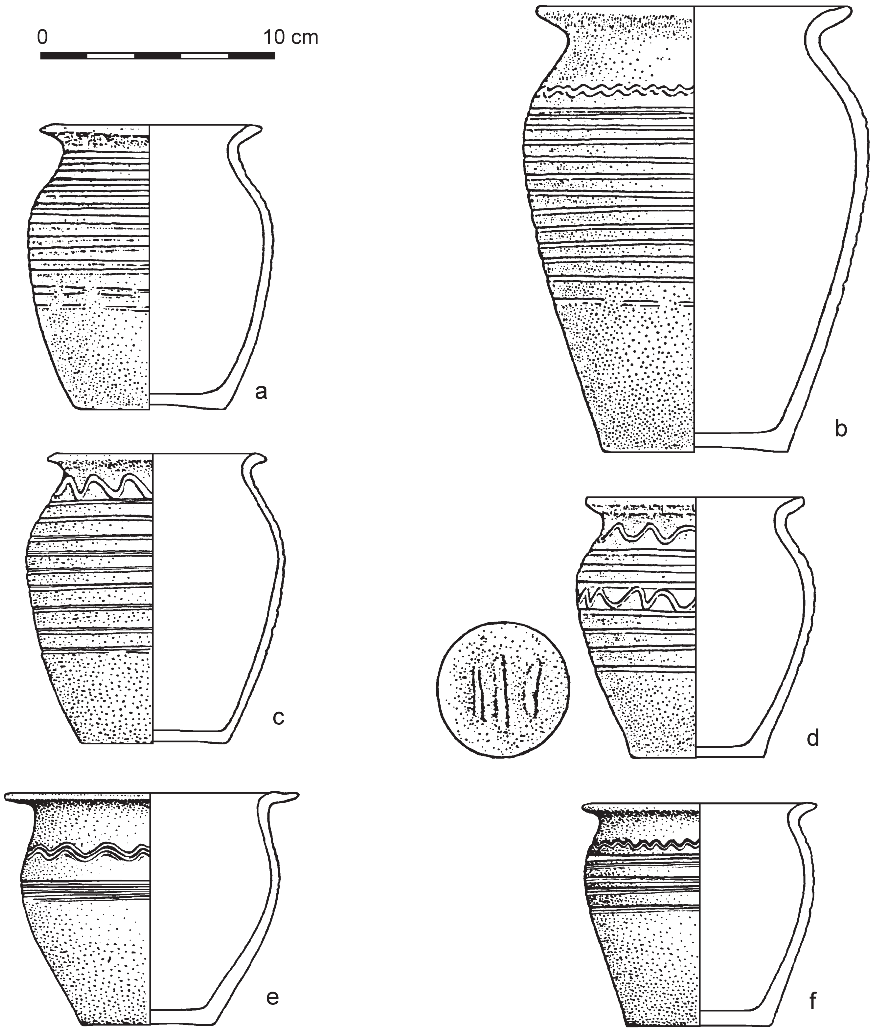

Fig. 10. Vessels from graves
a – grave 65; b – grave 60; c – grave 70; d – grave 68; e – grave 81; f – grave 83.
Drawing M. Pekáriková

Obr. 10. Nádoby z hrobov
a – hrob 65; b – hrob 60; c – hrob 70; d – hrob 68; e – hrob 81; f – hrob 83.
Rys. M. Pekáriková

is invariant to the size of the vessels examined is an advantage, making it possible to use for determining vessel shape similarity.

The main idea of the new method is as follows. Vessel shape is represented by a polygonal approximation of a set of consecutive line segments. The profile of a vessel is defined with a system of quantitative points as described in G. Fusek 1994, Fig. 11. Corner points on the object boundary were detected in order to obtain the

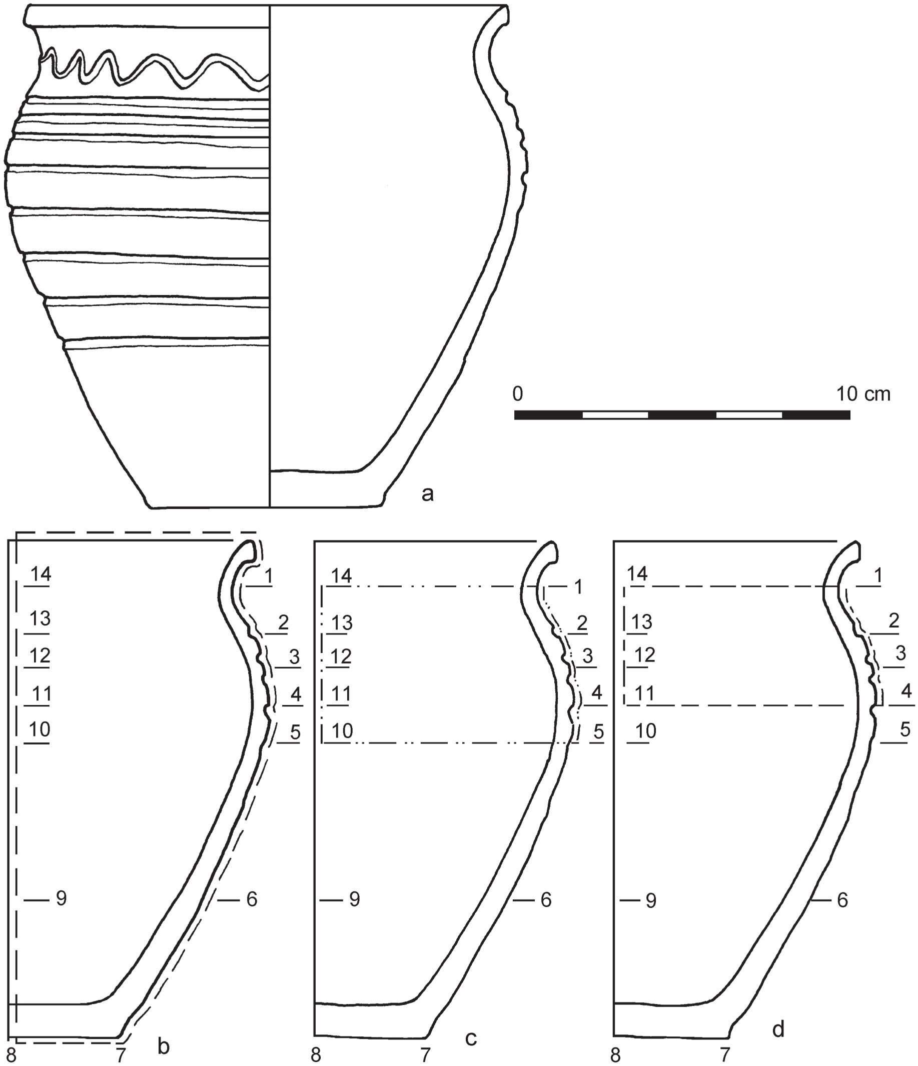

Fig. 11. Polygon for the form
a – main form from Nitra-Lupka; b – made up of seven pairs of points; c – made up of five pairs of points; d – made up of four pairs of points.

Drawing M. Pekáriková

Obr. 11. Polygón tvaru nádoby
a – hlavný tvar nádoby z Nitry-Lupky; b – polygón 7 párov bodov; c – polygón 5 párov bodov; d – polygón 4 párov bodov.

Rys. M. Pekáriková

polygonal representation (H. Ogawa 1994). Three polygons formed by line segments were selected (Fig. 11b-d). For example, a polygon (Fig. 11b) is formed by 14 line segments, which start from point 1, continue through point 14 and finish again in point 1 (closed polygon).

Let O_i be the line segment between the ith and the $(i+1)$th vertices of the polygon O. The polygon O can be represented by the ordered n-tuple of the line segments as follows:

In our case, we describe the similarity between two arbitrary line segments a and p in terms of their directions as well as in terms of the distance of their centres.

The similarity of line segments in terms of their direction

We will define the direction similarity between the two line segments a and p using the membership function $\mu_{ap}^{D} = |\cos(c_D \varphi_{ap})|^{n_D}$, $0 \le \varphi_{ap} \le \pi$, where φ_{ap} is the angle between the vectors a and p, n_D and c_D are the parameters. If two line segments have a similar height value of the membership function μ_{ap}^{D}, then corresponding vectors have a similar direction.

The similarity of line segments in terms of the distance of their centers

The distance between the locations of the line segment a and the line segment p is defined as a normalized distance between the middle points of two line segments. Without the loss of generality, we can assume that the coordinates are nonnegative numbers.

The location similarity between the two line segments a and p can be measured by the use of the membership function in the following form:

$$\mu_{ap}^{L} = \frac{1}{1 + e^{14.69x - 4.6523}}.$$

We note that F.H. Cheng, W.H. Hsu, C.A. Chen (1989) defined the membership function in the form $\exp(-t\varphi_{ab})$, where t is the parameter. The value of the parameter t is 0.012. Two line segments have a large value of the membership function, if they are close to one another. Afterwards, it was necessary to quantify the loss of information due to substitution of the line segment a by the line segment p (regarding the direction and location). If two line segments are considered to be the same object, then the information in this fuzzy object can be measured by the fuzzy entropy (C.E. Shannon 1948). This function is monotonous and its values are monotonically increasing within the interval [0,0.5] and monotonically decreasing in the interval [0.5,1]. As the function (marked as H^*) should be monotonically decreasing in the interval, i.e. to reflect the reality, the loss of information reaches maximum at the minimum value of the membership function μ ($\mu = 0$) and equals zero for $\mu = 1$ (F.H. Cheng, W.H. Hsu, C.A. Chen 1989). If we substitute μ by the value $0.5 + 0.5\mu$, then $H^*(\mu) = -(0.5 + 0.5\mu)\ln(0.5 + 0.5\mu) - (0.5 - 0.5\mu)\ln(0.5 - 0.5\mu)$.

Function H^* assumes the values from the interval $\langle 0, \ln 2 \rangle$ The value of the loss of information for matching line segment a and line segment p in terms of location similarity is denoted $H^*_{L_{ap}}$ and the value of the loss of information for matching line segment a and line segment p in terms of direction similarity is $H^*_{D_{ap}}$. The total value of the loss of information for matching line segment a and line segment p (when considering the direction and position together) is weighted by the sum $H^*_{T_{ap}} = w_L + H^*_{L_{ap}} + w_O + H^*_{D_{ap}}$, where w_L and w_O are the weights. The distance and the direction have equal weight, therefore the values equal $w_L = w_O = 0.5$. The total value of the loss of information for matching line segments $a_1, a_2, ..., a_n$ with line segments $p_1, p_2, ... p_n$ is

$$H^*_T = \frac{1}{n} \sum_{i=1}^{n} H^*_{T_{a_i p_i}}.$$

The function H^*_T assumes the values from the interval [0,1]. The more the two objects look alike, the lower the overall loss of information, i.e., the value of H^*_T is near to 0. In our case, $n = 14$ (Fig. 11b).

We can see that the function $1 - H^*_T$ indicates a fuzzy binary relation. A fuzzy binary relation which is reflexive, symmetric and transitive is called a relation of similarity. It is shown easily that $1 - H^*_T$ is a standard equivalence relation (G.J. Klir, Y. Bo 1995).

The classical relation of equivalence represents the similarity between elements on the degree α. The relation of similarity can be clearly illustrated as a "tree", whose nodes represent single equivalence classes. In this way, using the function $1 - H^*_T$, we can construct a hierarchical decomposition to the classes containing vessels similar in shape.

First the points from the whole profiles of vessels including the rims were analyzed, that is, the polygon with 7 pairs of points (Figs 11b and 12). The average value of several measurements of vessel profiles was considered. Based on the results of the fuzzy cluster analysis, it can be observed that 2 main groups of vessels were formed on the level 0.993 (Fig. 12). The results obtained in the study of 5 and 4 pairs of points (Fig. 11c, d) are very similar to those obtained through the analysis of 7 pairs of points. In all three cases, that is, the 7, 5 and 4 pairs of points, two main groups of vessels were obtained on the level 0.993 with the help of the new method. The first group includes the so-called wide vessels (Fig. 2a) and the second group includes the so-called slender vessels (Fig. 2b).

The vessels from the Nitra-Lupka locality were also grouped by the use of a standard cluster analysis. The cluster analysis was carried out for 4 groups of selected quantitative attributes, namely for: 1. measured quantitative variables of vessels (z1): overall height of the vessel, diameter of the rim, diameter of the neck, diameter of the maximum bulge, diameter of the bottom; 2. ratios of selected quantitative variables of vessels (z2): ratio of the overall height to the diameter of the maximum bulge; ratio of the diameter of the neck to the diameter of the maximum bulge; 3. quantitative variables (z12): cases 1 and 2 together; 4. selected quantitative variables (z3): overall height of the vessel, height of the rim, height of the shoulder and height from the bottom up to the maximum bulge.

Results obtained by fuzzy cluster analysis were compared to results obtained by standard cluster analysis. The objective was to see whether similar results would be obtained by standard cluster analysis for selected quantitative characters as by the fuzzy cluster analysis. In other words, we tested whether we would get the same clusters (groups) of vessels for any of these 4 groups of selected quantitative characters as we had obtained by the use of the fuzzy method.

The chi-square-test was used for a comparison of the two sets of results. This test allows the relationship between the two observed qualitative variables X and Y to be verified. In our case, the variable X denominates clusters obtained by the fuzzy

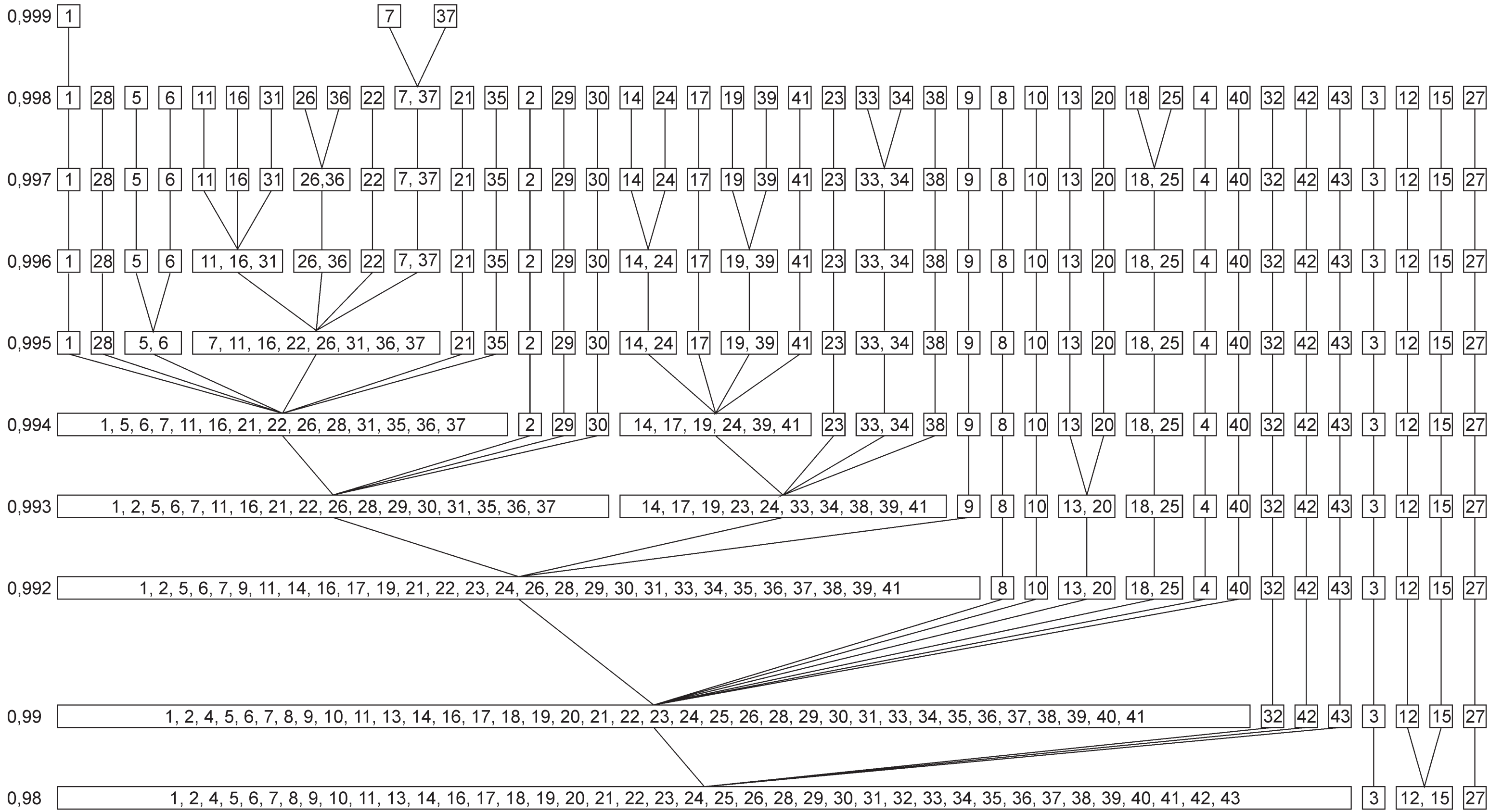

Fig. 12. Similarity diagrammed by a partition tree – made up of seven pairs of points.

Processing B. Stehlíková

Obr. 12. Dendrogram – rozhodovací strom podobnosti pre 7 párov bodov.

Vypracovala B. Stehlíková

cluster analysis and the variable Y is identified with standard clusters obtained by the cluster analysis. As a test criterion, we used the statistics given by the formula

$$\chi^2 = \sum_{i=1}^{k} \sum_{j=1}^{m} \frac{(f_{ij} - o_{ij})^2}{o_{ij}},$$

where f_{ij} represents the values of the empirical frequency and o_{ij} the values of the expected frequency. The test criterion χ^2 of the hypothesis H_0 validity has the chi-square-distribution with degrees of freedom $r = (k-1)(m-1)$. We reject the tested hypothesis H_0 at the significance level α, if the value of the test criterion χ^2 exceeds the critical value $\chi^2_\alpha(r)$.

In our case, we tested the following null hypothesis H_0: The clusters of vessels obtained by the fuzzy cluster analysis on the level 0.993 (ref. F1) or 0.994 (F2) are statistically significantly independent of the clusters obtained by the standard cluster analysis based on the selection of quantitative characters referred to in group z1 (and further in groups z2, z12 and z3).

The null hypothesis was contrasted with the alternative hypothesis H_1: The clusters of vessels obtained by the fuzzy cluster analysis on the level 0.993 or 0.994 are statistically significantly dependent on the clusters obtained by the standard cluster analysis based on the selection of quantitative characters referred to in group z1 (and further in groups z2, z12 and z3).

Calculations were made with the help of a computer using SAS (Statistical Analysis System). After entering the input data we got the value of the test criterion and the probability value p (p is the probability of an error committed, if we reject the tested hypothesis).

If the calculated probability value p is sufficiently small ($p < 0.05$, or $p < 0.01$), we reject the hypothesis about the independence of clusters (significance level is 0.05 or 0.01). The results for probabilities p are summarized in the following table (Table 1).

Table 1. Values of calculated probabilities
Tabuľka 1. Vypočítané hodnoty pravdepodobností

	F1 (0.993)	F2 (0.994)	z1	z2	z12	z3
F1 (0.993)		< 0.0001	0.2319	0.0021	0.2319	0.1599
F2 (0.994)	< 0.0001		0.4070	0.0152	0.4070	0.2338
z1	0.2319	0.4070		0.0178	< 0.0001	< 0.0001
z2	0.0021	0.0152	0.0178		0.0178	0.1677
z12	0.2319	0.4070	< 0.0001	0.0178		< 0.0001
z3	0.1599	0.2338	< 0.0001	0.1677	< 0.0001	

SUMMARY

The results obtained by using fuzzy cluster analysis and standard cluster analysis are compared in Table 1. As said already, the fuzzy cluster analysis resulted in two expected main groups of vessels being formed on the level 0.993 (Fig. 12).

The first group includes the so-called wide vessels (Fig. 2a), the second the so-called slender vessels (Fig. 2b).

Firstly, we want to the interpretation of the analysis results for the case in which the null hypothesis cannot be rejected. When comparing the results obtained by using fuzzy cluster analysis on the level 0.993 (F1) with the results received by standard cluster analysis based on the selection of quantitative variables described in z1, the calculated value of probability is $p = 0.2319$ (Table 1). It means that on the significance level $\alpha = 0.05$ we cannot reject the null hypothesis H_0, i.e., that the clusters of vessels obtained by fuzzy cluster analysis are independent (different) of the clusters of vessels we got within the standard cluster analysis.

Secondly, we show the case of rejection of the null hypothesis. When comparing the results obtained by fuzzy cluster analysis on the level 0.994 (F1, respectively F2) with results from a standard cluster analysis based on the selection of quantitative variables described in z2, the calculated value of probability is $p = 0.0152$ (Table 1). It means that on the significance level $\alpha = 0.05$ we reject the null hypothesis H_0, i.e., that the clusters of vessels obtained by fuzzy cluster analysis are similar (or identical) with the clusters of vessels identified by standard cluster analysis. Other results summarized in the table can be interpreted in the same way.

On the basis of a comparison of results obtained by two methods, the new fuzzy cluster analysis and the standard cluster analysis, we can conclude that the results are the same only in one case (z2). In other cases, standard cluster analysis produced groupings of vessels which had, for example, the same height or the same maximum bulge, etc., but the vessels within the clusters were of different shape.

It is thus concluded that standard cluster analysis on the basis of selected quantitative attributes produces clusters of vessels, but does not record vessel shape. In addition, standard cluster analysis is not invariant with respect to vessel size. Consequently, fuzzy cluster analysis is found to be appropriate for sorting vessels by shape.

From an archaeological point of view, it is remarkable that the ceramics from the graves and pottery kilns of Nitra-Lupka site have only exceptionally been recorded at other sites in the micro-region of former Nitra. The hillfort settlement has not been excavated, so one cannot comment on how many vessels the inhabitants had actually used. The size of the pottery kilns and their number indicate, however, that the production must have been much greater than what has been found in the graves. The kilns had been damaged significantly by deep ploughing, hence the collection of ceramics from the site was mostly fragmented and few complete pots could be reconstructed. The set from the cemetery, however, appeared to represent regular production and hence was chosen for verification of the new method. Two groups of vessels were distinguished by visual examination: a high and wide pot and a slender and usually smaller vessel. Analysis by the fuzzy cluster analysis not only grouped the vessels by shape into two major groups, but it also pointed to minor variations of shape of the studied vessels. These are displayed at various levels in a dendrogram. The results of the application of the new method demonstrated its suitability for finding similarly shaped vessels at other cemeteries as well.

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Abbreviations

"ŠzvAú" — "Študijné zvesti Archeologického ústavu Slovenskej akadémie vied v Nitre", Nitra.

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IVONA VLKOLINSKÁ, ANNA TIRPÁKOVÁ,
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UKÁŽKA NOVEJ METÓDY FUZZY MNOŽÍN PRE KLASIFIKÁCIU TVAROV NADOB Z POHREBISKA NITRA-LUPKA

R e s u m é

V článku je prezentovaná nová štatistickú metóda, ktorá zaznamenáva tvar nádob a triedi ich do skupín na základe ich tvaru. Pre porovnanie sú v článku uvedené aj niektoré vybrané kvantitatívne ukazovatele nádob (obr. 1). Analyzovaný súbor nádob bol rozdelený na dve skupiny. Do prvej skupiny patria tzv. široké nádoby (obr. 2a) a druhá skupina obsahuje tzv. štíhle nádoby (obr. 2b), pričom všetky analyzované nádoby sú zobrazené na obr. 3–10. Tvar nádob bol zaznamenaný pomocou polygónov (obr. 11) a na analyzovanie podobnosti tvarov nádob bola použitá nová metóda – fuzzy zhluková analýza.

V súlade s výsledkami získanými fuzzy zhlukovou analýzou možno pozorovať, že na úrovni 0,993 boli sformované dve predpokladané hlavné skupiny nádob (obr. 12). Výsledky, získané fuzzy zhlukovou analýzou sme porovnali s výsledkami, získanými štandardnou zhlukovou analýzou pomocou chi-kvadrát testu. Výsledky testovania sú prehľadne zapísané v tabuľke 1.

Uvedieme dva príklady interpretácie výsledkov porovnávania oboch metód zhlukovania nádob a to podľa ich tvaru (novou fuzza zhlukovou analýzou) a štandardnou zhlukovou analýzou na základe kvantitatívnych (metrických) vlastností nádob. Uvedieme príklad interpretácie výsledku, v prípade, že nemôžeme zamietnuť nulovú hypotézu, a tiež interpretácie výsledku v prípade, keď nulovú hypotézu zamietame (tabuľka 1).

Ak pri porovnaní výsledkov zhlukovania nádob oboma metódami pomocou chi-kvadrát testu nulovú hypotézu na hladine významnosti $\alpha = 0.05$ nemôžeme zamietnuť, to znamená, že zhluky nádob získané novou metódou sú odlišné (resp. nezávislé) od zhlukov získaných pomocou klasickej zhlukovej analýzy.

Ak pri porovnaní výsledkov zhlukovania nádob podľa ich tvaru oboma metódami pomocou chi-kvadrát testu nulová hypotézu na hladine významnosti $\alpha = 0.05$ zamietame, to znamená, že zhluky nádob získané novou metódou - fuzzy zhlukovou analýzou sú podobné (resp. identické) so zhlukmi získanými pomocou štandardnej zhlukovej analýzy.

Rovnakým spôsobom môžu byť interpretované aj ostatné výsledky, ktoré sú uvedené v tabuľke 1.

Na základe výsledkov chi-kvadrát testu, ktoré sme získali pri porovnávaní oboch metód zhlukovania nádob podľa ich tvaru, môžeme konštatovať, že výsledky získané novou metódou - fuzzy zhlukovou analýzou a štandardnou zhlukovou analýzou sú rovnaké len v jednom prípade a to v v prípade z2 (tabuľka 1).

Na základe týchto výsledkov sa tiež zistilo, že použitím klasickej zhlukovej analýzy na základe vybraných kvantitatívnych znakov nádob sme získali určité skupiny nádob, ktoré však nezohľadňujú ich tvar. To znamená, že klasická zhluková analýza nie je invariantná vzhľadom na veľkosti nádob, a preto triedenie nádob podľa ich tvaru pomocou klasickej zhlukovej analýzy môže byť nedostatočné.

Výsledky získané novou – fuzzy zhlukovou analýzou potvrdili, že táto nová metóda je vhodná na klasifikáciu nádob podľa ich tvaru. Na verifikovanie vhodnosti tejto metódy sme vybrali taký súbor nádob z pohrebiska, kde sú zastúpené tvarovo pomerne jednotné exempláre. Novou metódou sa potvrdilo rozdelenie nádob do dvoch očakávaných skupín. Aplikáciou novej metódy sme získali nielen rozdelenie do dvoch hlavných skupín, ale touto metódou sa tiež dosiahlo to, že boli zohľadnené niektoré menšie odlišnosti v tvare študovaných nádob. Tieto výsledky sú ilustrované na nasledujúcom dendrograme (Fig. 12).

Nová metóda umožňuje triediť nádoby podľa tvaru bez ohľadu na ich veľkosť, čo neumožňuje klasická zhluková analýza.

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