EFFECTIVE PROPERTIES OF MATERIALS WITH PERIODIC THIN WALLED CUBIC MICROSTRUCTURE

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1. Introduction

In this paper we study effective elastic properties of composites with a periodic microstructure that have a unit cell with geometrical cubic symmetry and such that one of its material phases builds up of thin plates. It is further assumed that the unit cell is made of two isotropic materials. An important example of such materials are foams. Certain microstructures, such as cubic–octet microstructure [1], give materials with the theoretical limit of elastic stiffness.

It is found in [2] that the homogeneous eigenstrain approximation to the equivalent eigenstrain principle gives an accurate prediction to the effective properties for certain thin walled structures. Among them are cubic, octet and cubic–octet microstructures. In fact, the approximation gives a closed form solution

(1)
$$\kappa_{\mathrm{e}} = 1 - \frac{3f(1-\kappa_{\mathrm{r}})(1-\nu)}{3(1-\nu)-(1+\nu)(1-\kappa_{\mathrm{r}})(1-f)},$$

(2)
$$\mu_{\mathrm{e}} = 1 - \frac{3f^2(1-\mu_{\mathrm{r}})(1-\nu)}{\Delta_1},$$

(3)
$$\hat{\mu}_{\mathrm{e}} = \frac{6f^2(1-\mu_{\mathrm{r}})^2(\nu-1)(5\beta(f)+(f-1)f)}{\Delta_1\Delta_2},$$

where

$$\Delta_1 = 2\beta(f)(1-\mu_{\mathrm{r}}) + 2f^2(1-\mu_{\mathrm{r}})(1-\nu) + f(2\mu_{\mathrm{r}}+1)(1-\nu),$$
$$\Delta_2 = -3\beta(f)(1-\mu_{\mathrm{r}}) + f^2(1-\mu_{\mathrm{r}})(1-2\nu) + f(\mu_{\mathrm{r}}(1-2\nu)-\nu+2).$$

Here κ_{r} and μ_{r} are the quotients of elastic moduli of material phases of the unit cell, ν is the Poisson ratio of the walls and f is the volume ratio between the materials of the unit cell with f near 1 for thin walled structures. Coefficient $\beta(f)$ which is obtained by solving the equivalent eigenstrain equation by the Fourier method is given by the series

(4)
$$\beta(f) = \sum_{\underline{m}\neq\underline{0}} \frac{c_0(\underline{m})^2\left(m_1^2m_2^2+m_1^2m_3^2+m_2^2m_3^2\right)}{|\,\underline{m}\,|^4}.$$

Here $c_0(\underline{m})$ is $\underline{m}$-nth Fourier coefficient of the compliment of the walls within the unit cell. It follows from (1-3) that the structural dependence of effective elastic properties depends only upon the function $\beta(f)$.

2. Elastic performance of composites

The elastic energy of composite subjected to the homogeneous macro strain : $\underline{\underline{\epsilon}}_0$ is given by the Hill–Mandel formula $\frac{1}{2}\underline{\underline{\epsilon}}_0 : \underline{\underline{C}}^{\mathrm{eff}} : \underline{\underline{\epsilon}}_0$. The elastic performance of the composite is measured by the amount of the elastic energy it can store. The amount is given by volume of the 6 dimensional ellipsoid with semi axes which are the eigenvalues of $\underline{\underline{C}}^{\text{eff}}$. They are $\lambda_1 = C^{\text{eff}}_{1212}$, $\lambda_2 = C^{\text{eff}}_{1111} - C^{\text{eff}}_{1122}$ $\lambda_3 = C^{\text{eff}}_{1111} + 2C^{\text{eff}}_{1122}$ with multiplicities 3, 2 and 1. The volume $\mathcal{V}$ is proportional to $\lambda_1^3\lambda_2^2\lambda_1$. Formulae (1-3) allow to express $\mathcal{V}$ as a function of ν, κ_{r}, μ_{r}, f and β. Regarding $\mathcal{V}$ as a function of β it is found that $\mathcal{V}(\beta)$ is stationary at five values of β. However, only one of them, namely $\beta = \frac{1}{5}(1-f)f$ depends solely on f, others depend also on ν and μ_{r}. It follows from (3) that at this value $\hat{\mu}_{\text{e}} = 0$. Therefore, the stiffness thin walled microstructure is isotropic.

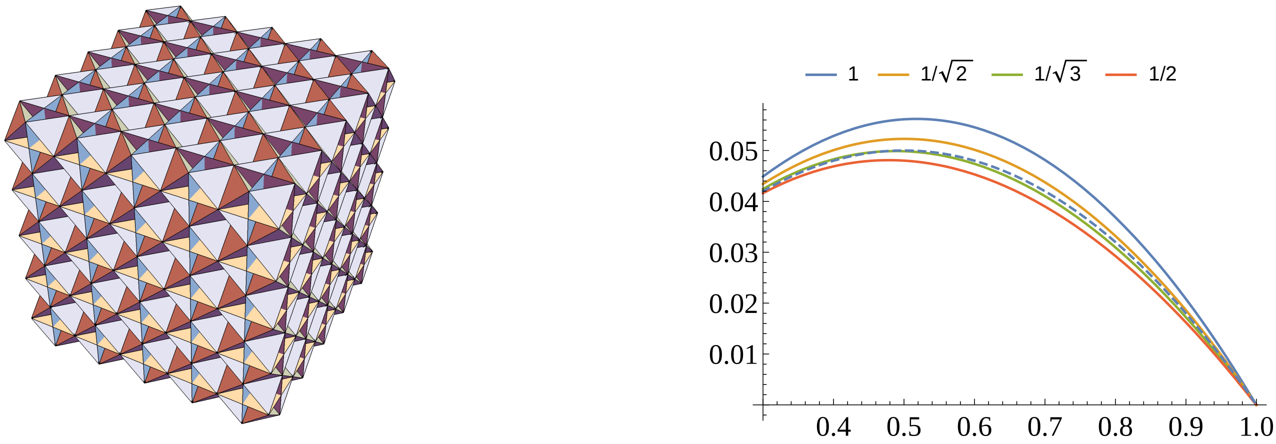

Figure 1: Cubic–octet foam and $\beta(f)$ for various ratios α of octet to coordinate walls thicknesses; from the bottom curve $\alpha = 1/2$ to the top curve with $\alpha = 2$. A dashed line is $\beta = \frac{1}{5}(1-f)f$.

It was shown in [1] by another method that the cubic–octet foam (see Figure 1), which is combination of the cubic and the octet structure, is for the ratio $1/\sqrt{3}$ of the octet to coordinate walls thicknesses isotropic. This is confirmed by the present results in Figure 1. Moreover, not only cubic–octet foam but any composite with thin walls of the cubic–octet structure is macroscopically isotropic. Of course, function $\beta(f)$ does not unequally determines shape of the unit cell. It is aim of the further research to identify other thin walled structures that are isotropic. On the other hand, dependence of elastic moduli upon β allows also to formulate conditions on β for other extremal elastic properties such as extremal values of the Poisson's or Zener ratios.

References

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