SYSTEMS RESEARCH INSTITUTE
POLISH ACADEMY OF SCIENCES

INTERNATIONAL INSTITUTE FOR APPLIED SYSTEMS ANALYSIS

CONTRACTED STUDY AGREEMENT REG / POL / 1

"CONCEPTS AND TOOLS FOR STRATEGIC REGIONAL SOCIO-ECONOMIC CHANGE POLICY"

STUDY REPORT

PART 1

BACKGROUND METHODOLOGIES

COORDINATOR, IIASA: A. KOCHETKOV
COORDINATOR, SRI PAS: A.STRASZAK

ZTS/ZPZC/ZTSW 1-36/85

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Consisting of 3 Parts

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VII. STRATEGIC ASPECTS OF TAX REGULATIONS

by J. Stefański and A. Straszak

VII.1 Introduction

Some aspects of the complex nature of socio-economic processes taking place in a region can be satisfactorily explained if the existence of a multiplicity of decision makers optimizing different objectives and making different decisions is taken into account. An example is a regional economy system in which a center (a government) influences the behavior of economic agents (Salman, Cruz, 1981; Stefański, Cichocki, 1985; Takayama, Simaan, 1983).

In this chapter we formulate a model of a regional economy system in which a center controls the enterprises´ behavior through the use of a specific tax system. This system has been designed by the Polish governmental agencies for the possible use in the coming years; currently the effects it may have are under consideration (Straszak and others, 1985; see also Straszak, Stefański, Ziółkowski - this volume, Chapter IX). The situation is modelled as a dynamic game with a center and several groups of firms as players. There are two aspects of the game worth emphasizing. First, the nonsymmetric statuses of the parties, which can be interpreted as a hierarchy in the game. There are various reasons for introducing such a hierarchy in an economy system (Auger, 1985) but we take into account only the roles´ differentiation (between a center and enterprises) which follows from the tax system. The second aspect of the game we concentrate on is the possibility(that the tax system offers) of making a center-firm agreement about the conditions bound up with the use of reduced rate tax regulations. As reality suggests we assume that such an agreement is not absolutely binding for the firm and therefore the center prevents its violation by the threat of using a retaliation tax strategy. The idea of reaching, in a dynamic game, a cooperative agreement which is not formally binding has been discussed by Tołwiński (1982) and Haurie and Tołwiński (1984).

They considered a two-person game with symmetric roles of the players and defined a set of cheat-proof equilibrium strategies in the class of memory strategies. In our case the nonsymmetry of the parties´statuses and the existence of Nash equilibrium at the lower (firms´) level modify the problem.

VII.2 Regional economy system as a dynamic game

We take into account a regional economy system in which we distinguish a center and M groups of enterprises (see Fig. 1). The aggregation of enterprises into groups has been made on the basis of their costs´structure, situation on the sales market (equilibrium or disequilibrium, the shape of the demand function in the former case), etc. In the sequel we will treat each such group like a single firm.

In the model attention is focussed on the ways in which the center can influence the firm´s behavior using a tax mechanism bound up mainly with the increase of wage funds. And therefore we neglect, for simplicity, the competition among firms on sales markets. There are two reasons for concentrating on the wage / labour /production aspects of the firm´s activity. First, it has turned out that the decisions made in a firm result from the labour-management game (Chen, Leitman, 1980; Stefański, 1985a, 1985b), and second, the authorities in Poland attach much importance to the aim of bringing down inflation. And one of the main tools for doing this is the tax system we take into account in the model.

Decisions, Dynamics

Each firm (a group of firms) i, $i \in \{1,2...,M\}$, makes in each time period $t \in \{1,2,...,T\}$ the decisions concerning: production level $q_i(t)$, wage fund $w_i(t)$, and employment $y_i(t)$. Those decisions must belong to a certain admissible set U_i^t which we will now determine.

If we denote the minimal admissible individual wage by $v_{min} > 0$, then there must be $w_i(t) > v_{min} y_i(t)$. On the other hand the maximal number of persons an enterprise is in a position to employ depends on its wage competitiveness and the attractiveness of the work itself. We assume that those dependences are described by a function h_i, which is different for each firm

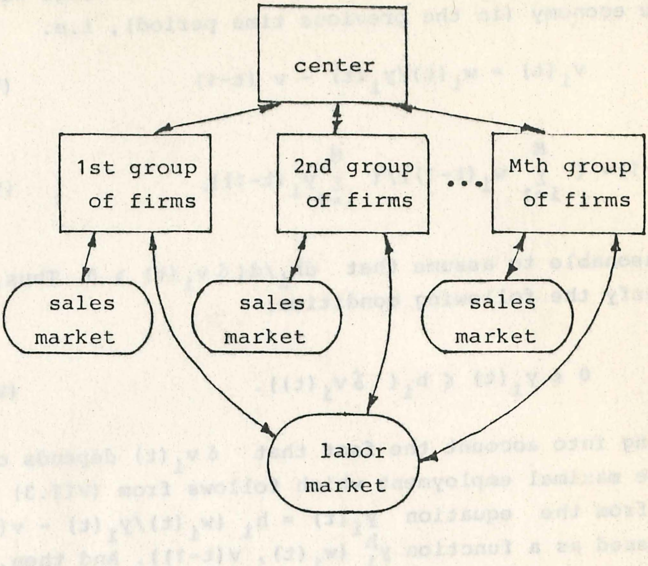

Fig. VII.1. General structure of the model

$i \in \{1,2,...,M\}$ and as an argument has the deviation $\delta v_i(t)$ of the average wage in the i-th firm from the average wage in the whole economy (in the previous time period), i.e.

$$v_i(t) = w_i(t)/y_i(t) - v(t-1) \qquad \text{(VII.1)}$$

where

$$v(t-1) = \left(\sum_{i=1}^{M} w_i(t-1)\right) / \left(\sum_{i=1}^{M} y_i(t-1)\right). \qquad \text{(VII.2)}$$

It is reasonable to assume that $dh_i/d(\delta v_i(t) \geq 0$. Thus, $y_i(t)$ must satisfy the following condition:

$$0 \leq y_i(t) \leq h_i(\delta v_i(t)). \qquad \text{(VII.3)}$$

Taking into account the fact that $\delta v_i(t)$ depends on $y_i(t)$, the maximal employment which follows from (VII.3) can be obtained from the equation $y_i(t) = h_i(w_i(t)/y_i(t) - v(t-1))$ and expressed as a function $y_i^h(w_i(t), v(t-1))$. And then, taking into account the previously mentioned constraint v_{min} we can determine the function y_i^m which determines the maximal possible and admissible employment level:

$$y_i^m(w_i(t), v(t-1)) = \min\{y^h(w_i(t), v(t-1)),$$

$$w_i(t)/v_{min}\}. \qquad \text{(VII.4)}$$

The next decision, production $q_i(t)$, can reach its maximal level $q_i^m(t)$ which depends on the possibilities described by a production function P_i which, in turn, depends on employment $y_i(t)$ and capital $x_i(t)$, i.e. we obtain the range from which $q(t)$ must be taken: $0 \leq q_i(t) \leq P_i(x_i(t), y_i(t))$ (where P_i can be, for instance, in the form of a Cobb-Douglas production function).

The vector of the i-th firm's decisions in period $t \in \{1,2,...,T\}$ will be denoted by:

$$u_i(t) = (q_i(t), y_i(t), w_i(t)) \qquad \text{(VII.5)}$$

where, taking into account the previous considerations:

$$u_i(t) \in U_i^t = \{ (q,y,w) : w \geqslant v_{min}\, y \,, \qquad \text{(VII.6)}$$

$$0 \leqslant y \leqslant y^m(w,v(t-1)) \,,\; 0 \leqslant q \leqslant P_i(x_i(t),\, y) \,\}$$

i=1,2...,M. We assume that $v(0)$ is given.

The dynamics of a firm is described by the changes of the capital $x_i(t)$ which is treated as a state variable:

$$x_i(t+1) = x_i(t) + \pi_i(x_i(t),\, u_i(t)) - u_{oi}(t), \qquad \text{(VII.7)}$$

where $\pi_i(\cdot)$ is the profit defined in the following way:

$$\pi_i(x_i(t),u_i(t)) = q_i(t)p_i(q_i(t)) - C_i(x_i(t),\, q_i(t),\, y_i(t)) - w_i(t), \qquad \text{(VII.8)}$$

where, in turn, $p_i(\cdot)$ is the price (which follows from the demand in the case of market equilibrium or is determined in another way in the case of market disequilibrium), and C_i is a cost function (excluding wages taken into account separately). The variable $u_{oi}(t) \geqslant 0$ in (VII.7) is the center´s decision variable. It is the tax which must be paid by the i-th firm in period t ($u_{oi}(t)$ follows from the specific tax system we have incorporated in the model, all other tax liabilities, e.g. profit tax, are neglected). In general, the state equations (VII.7) can be written in the following way:

$$x_i(t+1) = f_i(x_i(t),\, u_{oi}(t),\, u_i(t)), \qquad \text{(VII.9)}$$

$$t \in \{1,2,\dots,T\}\,,\; i=1,2,\dots,M.$$

If a firm is in state $x_i(t)$, the future trajectory $x_i(t+1)$, $x_i(t+2),\dots,x_i(T)$ depends on $x_i(t)$ and the firm´s and center´s control sequences:

$$\tilde{u}_i^t \overset{\Delta}{=} (u_i(t),\, u_i(t+1),\dots,u_i(T)), \qquad \text{(VII.10)}$$

$$\tilde{u}_{oi}^t \overset{\Delta}{=} (u_{oi}(t),\, u_{oi}(t+1),\dots,u_{oi}(T)). \qquad \text{(VII.11)}$$

Objectives

When choosing the sequences (VII.10) or (VII.11) the decision makers want to maximize the following stage-additive objective functions, for firms:

$$G_i\,(t, x_i(t),\, \tilde{u}_{oi}^t,\, \tilde{u}_i^t) \overset{\Delta}{=} \sum_{s=t}^{T} g_i(x_i(s),\, u_{oi}(s),\, u(s)), \qquad \text{(VII.12)}$$

$i=1,2,\ldots,M,$

and for the center:

$$G_o(t, x(t),\, \tilde{u}_o^t,\, \tilde{u}^t) \overset{\Delta}{=} \sum_{s=0}^{T} g_o(x(s), u_o(s), u(s)), \qquad \text{(VII.13)}$$

where

$$x(t) = (x_1(t),\, x_2(t), \ldots, x_M(t)),$$

$$\tilde{u}_o^t = (\tilde{u}_{o1}^t,\, \tilde{u}_{o2}^t, \ldots,\, \tilde{u}_{oM}^t),$$

$$\tilde{u}^t = (\tilde{u}_1^t,\, \tilde{u}_2^t, \ldots,\, \tilde{u}_M^t), \qquad \text{(VII.14)}$$

$$u_o(t) = (u_{o1}(t),\, u_{o2}(t), \ldots, u_{oM}(t)),$$

$$u(t) = (u_1(t),\, u_2(t), \ldots, u_M(t)).$$

The form of the functions g_i which appear in (VII.12) are assumed to be linear combinations of three parts:

$$g_i(x_i(t), u_{oi}(t), u_i(t)) =$$
$$= \alpha_1(\pi_i(x_i(t), u_i(t)) - u_{oi}(t)) + \qquad \text{(VII.15)}$$
$$+ \alpha_2 \beta_t w_i(t) + \alpha_3 \beta_t w_i(t)/y_i(t),$$

$i=1,2,\ldots,M$, where $\alpha_1, \alpha_2, \alpha_3 \geqslant 0$ and

$$\beta_t = \prod_{s=1}^{t} (1 - \varrho_t), \qquad \text{(VII.16)}$$

where ϱ_t is the inflation rate forecasted for the year t. The first component of (VII.15) is the profit minus tax liability, the second reflects the aim of maximizing the employees´income, while the last part - the aim of maximizing the average wage. As follows from the above definition of g_i it reflects the compromise which combines the interests of employees (the second and third parts) and of the firm's management (the first component) (Stefański, 1985a, 1985b).

Function g_o which appears in (VII.13) has also the form of a linear combination of various center's goals:

$$g_o(x(t), u_o(t), u(t)) = \alpha_{o1} \sum_{i=1}^{M} u_{oi}(t) + \alpha_{o2} \sum_{i=1}^{M} \ell_i q_i(t) + \alpha_{o3} \sum_{i=1}^{M} \ell_i q_i(t) / \sum_{i=1}^{M} w_i(t) + \alpha_{o4} \sum_{i=1}^{M} y_i(t) , \qquad (VII.17)$$

where $\alpha_{o1}, \ldots, \alpha_{o4} \geqslant 0$, and ℓ_i, $i=1,2,\ldots,M$, are the multipliers which reflect social utility of production of each firm (the effect is as if the production were expressed in the same units). The first component in (VII.17) is the sum of taxes paid in a period, the second leads to maximization of production, the third part reflects the aim of reducing the inflation rate (which, in Polish conditions, can be attained by maximizing the ratio of global production to global incomes). The last part in (VII.17) represents the aim of maximization of employment.

<u>Tax system, information structure, strategies</u>

To complete the description of the model structure we must specify the way in which decision are made by the parties, and the information they have access to. It will allow us to consider the strategies (decision rules).

Let us recall that the center has at each stage $t \in \{1,2,\ldots,T\}$, M decision variables $u_{oi}(t)$, $i=1,2,\ldots,M$, at its disposal. The value $u_{oi}(t)$, which is the tax liability of the i-th firm, is the realization of a center's strategy, i.e. $u_{oi}(t) = \gamma_{oi}(\cdot)$.

The way in which this strategy is determined follows from the tax system incorporated in the model. The characteristic feature the system has is that it allows negotiations between the center and a firm on the condition of application of preferential (reduced rate) tax regulations.

The tax system under consideration is based on K + 1 tax formulas (in the original system K+1=5) which form the set

$$H = \{\eta_o, \eta_1, \ldots, \eta_K\}. \qquad \text{(VII.18)}$$

Each of these formulas (see Straszak and others, 1986) is a mapping

$$\eta_k : (x_i(t), u_i(t), u_i(t-1)) \rightarrow R_+ \qquad \text{(VII.19)}$$

$$k \in \{0,1,\ldots K\},$$

$i \in \{1,2,\ldots,M\}$, $t \in \{1,2,\ldots,T\}$ (we assume that the values $u_i(0)$ are given). All the parties taking part in the game know the set H. The formula η_o is called a <u>basic formula</u>, and formulas $\eta_1, \eta_2, \ldots, \eta_K$ are called <u>preferential</u> or <u>reduced rate</u> <u>formulas</u>. In a typical situation all firms work under the basic formula η_o, whereas the preferential formulas can be obtained on the basis of the individually negotiated agreements. As a result of such an agreement a firm is obliged to satisfy certain specified conditions concerning its activity. The problem of negotiated agreements will be discussed in another section, and now we only mention that it results in applying memory strategies.

As reality suggests we assume that the roles of the center and the firms are not symmetric, since the center´s strategy, i.e. the tax regulation under which a firm will be working in a particular year must be known to it at the beginning of the year, i.e. before the firm makes its decisions. The center´s strategy is then implemented at the end of a year (i.e. when the center knows the firm´s decisions and their results). Moreover, the players remember all the past decisions. In other words the closed loop with memory information structure and decision sequence is such that center can be thought to be the leader of the game. Thus, the information the center has access to in time period t is:

$$Z_o^t = (x(1);\gamma_o^1, \gamma_o^2, \ldots, \gamma_o^{t-1}; \quad u(1), u(2), \ldots, u(t)), \qquad \text{(VII.20)}$$

$t \in \{1,2,\ldots,T\}$, where

$$\gamma_o^t = (\gamma_{o1}^t, \gamma_{o2}^t, \ldots, \gamma_{oM}^t), \qquad \text{(VII.21)}$$

$$u(t) = (u_1(t), u_2(t), \ldots, u_M(t)).$$

On the other hand the i-th firm´s information set contains:

$$Z_i^t = (x(1);\gamma_o^1, \gamma_o^2, \ldots, \gamma_o^t; \quad u(1), u(2), \ldots, u(t-1), \qquad \text{(VII.22)}$$

$t \in \{1,2,\ldots,T\}$, $i=1,2,\ldots,M$.

Then the center´s strategy concerning the i-th firm is a sequence $\gamma_{oi} = (\gamma_{oi}^t)_{t=1,2,\ldots,T}$ of mappings:

$$\gamma_{oi}^t : Z_o^t \to R_+ \qquad \text{(VII.23)}$$

$i=1,2,\ldots,M$, and its realizations are $u_{oi}(t) = \gamma_{oi}^t(\cdot)$. On the other hand the i-th firm´s strategy is a sequence $\gamma_i = (\gamma_i^t)$, $t=1,2,\ldots,T$, of mappings:

$$\gamma_i^t : Z_i^t \to U_i^t, \qquad \text{(VII.24)}$$

where the set U_i^t of admissible decisions is defined by (VII.6). Because the problem is deterministic and players have information about the system´s dynamics, they can, on the basis of the information they recall, reconstruct the trajectory $(x(1), x(2), \ldots, x(t)$ of the system.

VII.3 Basic strategies

By basic strategies we mean the firm´s strategies in the situation in which all firms work under the basic tax regulation η_o. Let us recall that the firms in the model under consideration interact with each other only through labor market (the competition on sales markets is neglected). These dependencies are indirect because other firms influence in fact only the set U_i^t of admissible decisions of the i-th firm (this set depends on the average wage v(t-1), see (VII.6), (VII.2). In order to emphasize this we will write:

$$u_i(t) \in U_i^t(x_i(t), u(t-1)), \qquad \text{(VII.25)}$$

where $u(t-1) = (u_1(t-1),\ldots,u_M(t-1))$. Then, a sequence of decisions $\tilde{u}_i^t = (u_i(t), u_i(t+1),\ldots u_i(T))$ is admissible if it is an element of

$$\tilde{U}_i^t = \prod_{s=t}^{T} U_i^s\ (x_i(s), u\ (s-1))\ , \qquad \text{(VII.26)}$$

where

$$x_i(s) = f_i(x_i(s-1), u_{oi}(s-1), u_i(s-1)) \qquad \text{(VII.27)}$$

Strategies $(\gamma_i^*)_{i=1,2,\ldots,M}$ which generate the control sequences u_i^{1*}, i=1,2,...,M, form an <u>equilibrium</u> in the game with given x(1) and u(0), if all subsequences $\tilde{u}_i^{t*}$ of $\tilde{u}_i^{1*}$ for $t\in\{1,2,\ldots,T\}$, i=1,2,...,M, satisfy the following conditions:

$$\tilde{u}_i^{t*} = \arg\max_{\tilde{u}_i^t \in \tilde{U}_i^{t'}} G_i\ (t, x_i^*\ (t), \tilde{u}_{oi}^t, \tilde{u}_i^t), \qquad \text{(VII.28)}$$

where

$$\tilde{U}_i^{t'} = U_i^t\ (x_i^*(t), u^*\ (t-1))\ \mathrm{X} \prod_{s=t+1}^{T} U_i^s(x_i(s), u(s-1)), \qquad \text{(VII.29)}$$

where $u(s) = (u_1(s),\ldots,u_M(s))$ and

$$u_j(s) = \gamma_j^*(Z_j^s), \quad j\neq 1\ , \quad s=t, t+1,\ldots,T, \qquad \text{(VII.30)}$$

$$Z_j^s = (x(1);\eta_o;u^*(1),\ldots,u^*(t-1), u(t),\ldots,u(s-1));$$

moreover:

$$u_{oi}(t) = \eta_o(x_i^*\ (t), u_i(t), u_i^*\ (t-1)),$$
$$\qquad \text{(VII.31)}$$
$$u_{oi}(s) = \eta_o(x_i(s), u_i(s), u_i(s-1))$$

for s=t+1, t+2,...,T,
where

$$x_i(t+1)=f_i(x_i^*(t), u_{oi}(t), u_i(t)), \qquad \text{(VII.32)}$$

$$x_i(s) = f_i(x_i(s-1),u_{oi}(s-1),u_i(s-1))$$

for s=t+2,t+3,...,T,

and if $t \in \{2,3,...,T\}$ then

$$\begin{aligned} x_i^*(t) &= f_i(x_i^*(t-1),u_{oi}^*(t-1),u_i^*(t-1)), \\ u_{oi}^*(t-1) &= \eta_o(x_i^*(t-1), u_i^*(t-1), u_i^*(t-2)) \end{aligned} \qquad \text{(VII.33)}$$

Conditions (VII.28)-(VII.33) mean that along the optimal trajectory the principle of optimality must be satisfied. The above equilibrium is a Nash equilibrium in our game. The characteristic feature of the situation is that other firms influence the problem (VII.28) connected with the i-th firm only through the changes of the shape of the set of admissible decisions.

In the model we assume that the tax regulation (center's strategy) $\eta_o \in H$ is given. Note, that if we considered the problem of the optimal design of η_o, we should obtain a Stackelberg game problem (Basar, Olsder, 1982; Ho, Luh, Olsder, 1982; Zheng, Basar, Cruz, 1984; Zheng, 1984).

VII.4 Negotiated agreements

As we said earlier one of the preferential formulas $\eta_k \in H \setminus \{\eta_o\}$ can be applied only on the basis of an i-th firm-center agreement. Any $\eta_k \in H \setminus \{\eta_o\}$ is advantageous because

$$\eta_k(x_i(t),u_i(t),u_i(t-1)) \leq \eta_o(x_i(t),u_i(t),u_i(t-1)) \qquad \text{(VII.34)}$$

for $k \in \{1,2,...,K\}$, $i \in \{1,2,...,M\}$, $t \in \{1,2,...,T\}$.
But in return for applying η_k (instead of η_o) the center can oblige a firm to fulfil certain conditions (e.g. $\pi_i(x_i(t), u_i(t)) \geq \pi^t$, or $w_i(t)/q_i(t) \leq a^t$, or $y_i(t) \leq a_y^t$, or $q_i(t) \geq a_q^t$, where π^t, a^t, a_y^t, a_q^t are given values). In general we write

that in the following way:

$$F_i^t\,(x_i(t),u_i(t))\in \Phi_i^t\ ,\ t\in\{1,2,\dots,T\}. \qquad \text{(VII.35)}$$

The particular preferential formula $\eta_k \in H\setminus\{\eta_o\}$ as well as functions $\{F_i^t\}_{t=1,2,\dots,T}$ and sets $\{\Phi_i^t\}_{t=1,2,\dots,T}$ are the subjects of negotiations between the center and the i-th firm. We assume that only agreements for the whole period $\{1,2,\dots,T\}$ are possible. But it would be unrealistic if we assumed that a firm would never want to break an agreement (if it would be advantageous). And therefore in order to make agreement last the center incorporates in it a clause which, in fact, has a character of a retaliation threat. To be more precise, if the agreement is broken (i.e. $F_i^t\,(\cdot)\notin \Phi_i^t$) then the center returns to the basic regulation η_o and imposes a financial penalty ϕ_i^t on the firm. The center's <u>retaliation strategy</u> in the form $\eta_o(\cdot)+\phi_i^t$ is applied during a pre-specified number τ_i^R of time periods. Thus, we can define the <u>center's strategy</u> concerning the i-th firm as a sequence $\gamma_{oi} = (\gamma_{oi}^t)_{t=1,2,\dots,T}$, where the mapping γ_{oi}^t are defined in the following way:

$$u_{oi}(t) = \gamma_{oi}^t\,(\cdot) =$$

$$= \begin{cases} \eta_k(x_i(t),u_i(t),u_i(t-1) \ \text{if}\ F_i^s\,(x_i(s),\,u_i(s))\in\Phi_i^s \\ \qquad\qquad \text{for}\ s\in\{t-\tau_i^R, \qquad \text{(VII.36)} \\ \qquad t-\tau_i^R+1,\dots,t\}\ , \\ \eta_o(x_i(t),\,u_i(t),\,u_i(t-1)) + \phi_i^t \quad \text{otherwise} \end{cases}$$

where $\eta_k \in H\setminus\{\eta_o\}$, $\phi_k^t \geqslant 0$.

Note that retaliation during a limited time period τ_i^R gives the firm a chance to return to respecting the agreement (the other possibility is that the act of breaking an agreement makes it impossible to return to a cooperative mood of play-like in the model described in Haurie, Tolwinski, 1984).

Note, also, that the nonsymmetry of the players´ statuses and the sequence of decisions bound up with it result in the fact that retaliation takes place in the same period when the agreement is broken (Haurie and Tolwiński (1984) consider another case when cheating for one stage is possible).

In the sequel we outline the way of reaching a lasting agreement. Because of the lack of space we confine our interests to the case when only one firm makes an agreement with the center (the other work under η_o).

Let us define first the set of the firm´s decisions which are admissible and secure respecting of the agreement given by $F_i = (F_i^t)_{t=1,2,\dots,T}$ and $\Phi_i = (\Phi_i^t)_{t=1,2,\dots,T}$ (see (VII.35)). For $t \in \{1,2,\dots,T\}$ we have:

$$W_i^t\,(F_i^t,\,\Phi_i^t,\,x_i(t),\,u_i(t-1)) = \tag{VII.37}$$

$$= \{u_i(t) \in U_i^t(x_i(t),\,u_i(t-1)\;:$$

$$F_i^t\,(x_i(t),\,u_i(t)) \in \Phi_i^t\}\;.$$

We will say that an agreement based on F_i and Φ_i is <u>feasible</u> if for all $t \in \{1,2,\dots,T\}$ the sets (VII.37) are not empty, i.e.

$$W_i^t\,(F_i^t,\,\Phi_i^t,\,x_i(t),\,u_i(t-1)) \neq \emptyset\,, \tag{VII.38}$$

$$t = 1,2,\dots,T.$$

We are now in a position to define the corresponding set of decision sequences $\tilde{u}_i^1$ which are <u>admissible and respect the agreement</u> based on F_i and Φ_i:

$$W_i\,(x(1),\,F_i,\,\Phi_i) =$$

$$= \prod_{s=1}^{T} W_i^s(F_i^s,\,\Phi_i^s,\,x_i(s),\,u_i(s-1)) \tag{VII.39}$$

Let us now take into account an agreement and the corresponding center´s strategy $\gamma_{oi} = (\gamma_{oi}^t)_{t=1,2,\dots,T}$. In order to determine an equilibrium at the lower level in such a case we must replace η_o by γ^t_{oi} in (VII.31) and (VII.33) when computing $u_{oi}(t)$. The control sequences referring to the equilibrium obtained will be denoted by: $\tilde{u}^{1A}_i$ for the i-th firm (which the agreement negotiated), and by $\tilde{u}^{1ai}_j$ for $j \neq i$ (other firms). These control sequences, as well as the outcomes of the game depend on the attained agreement defined by the vector:

$$\psi_i = (\eta_k, F_i, \phi_i, \phi_i, \tau^R_i), \qquad \text{(VII.40)}$$

where $\eta_k \in H \setminus \{\eta_o\}$, $F_i = (F^t_i)_{t=1,2,\ldots,T}$, $\phi_i = (\phi^t_i)_{t=1,2,\ldots,T}$, $\phi_i = (\phi^t_i)_{t=1,2,\ldots,T}$. In order to emphasize the above mentioned dependence we denote the outcomes:

$$z_o = J_o(\psi_i) \triangleq G_o(1, x(1), (\tilde{u}^{1A}_{oi}, \tilde{u}^{1ai}_{oj}), (\tilde{u}^{1A}_i, \tilde{u}^{1ai}_j)), \qquad \text{(VII.41)}$$

$$z_i = J_i(\psi_i) = G_i(1, x(1), \tilde{u}^{1A}_{oi}, \tilde{u}^{1A}_i), \qquad \text{(VII.42)}$$

where

$$u^A_{oi}(t) = \gamma^t_{oi}(\cdot), \; u^{ai}_{oj}(t) = \eta_o(\cdot), \; j \neq i. \qquad \text{(VII.43)}$$

Let us introduce an argument when writing $\tilde{u}^{1A}_i(\psi_i)$ in order to emphasize the dependence of this decision sequence on the agreement. We will define now the set Ψ_i of all feasible and lasting agreements:

$$\Psi_i = \{\psi_i : \tilde{u}^{1A}_i(\psi_i) \in W_i(x(1), F_i, \phi_i) \neq \emptyset\}, \qquad \text{(VII.44)}$$

where $\psi_i = (\eta_k, F_i, \phi_i, \phi_i, \tau^R_i)$. Note, that $\tilde{u}^{1A}_i(\psi_i)$ together with $\tilde{u}^{1ai}_j$, $j \neq i$, form the equilibrium solution at the lower level defined by the set of equations analogous to (VII.28)-(VII.33). Thus, $\tilde{u}^{1A}_i(\psi_i) \in \tilde{W}_i(\cdot) \subset U^1_i$ means that this decision sequence is not only admissible but it also respects the agreement ψ_i (see definition (VII.39)). In other words the agreement ψ_i will be lasting since the firm has no incentive to break it. The second condition in (VII.44), i.e.

$W_i(\cdot) \neq \emptyset$ means that the agreement ψ_i is feasible (see VII.38)).

Now we are in a position to define the set of outcomes associated with all feasible and lasting agreements:

$$S_i = \{(z_o, z_i) : z_o = J_o(\psi_i), z_i = J_i \psi_i) \quad \text{for all } \psi_i \in \Psi_i\}, \qquad \text{(VII.45)}$$

where J_o, J_i are given by (VII.41), (VII.42) and Ψ_i by (VII.44). The bargaining problem the center and i-th firm face consists in finding such an agreement $\hat{\psi}_i \in \Psi_i$ that the associated pair of outcomes (z_o, z_i) will satisfy the both parties. As a status quo, i.e. a point of departure in negotiations, it seems resonable to take the outcomes (z_o^*, z_i^*) which are the outcomes resulting from the equilibrium under η_o (i.e.without a center-firm cooperation). Thus we have the (static) bargaining game defined by:

$$(S_i, (z_o^*, z_i^*)) \qquad \text{(VII.46)}$$

The classical Nash's method of solving a bargaining problem (Nash, 1950; Roth, 1979) requires the convexity of S_i (similarly as the solution suggested by Yu (1973)). Other methods (e.g. Kalai, Smorodinsky, (1975), Stefański, (1985b)) require the Pareto-frontier $P(S_i)$ to be a connected set. But, as pointed out by Haurie and Tołwiński (1984), we cannot assume that the sets like S_i are convex or even connected. They suggested a solution for bargaining problems when the sets of outcomes are compact only. Let us denote that solution of (VII.46) by $V(S_i, (z_o^*, z_i^*))$. Thus, the outcomes resulting from an agreement we were looking for are:

$$(\hat{z}_o, \hat{z}_i) = V(S_i, (z_o^*, z_i^*)) \qquad \text{(VII.47)}$$

Then, taking into account that

$$\hat{z}_o = J_o(\hat{\psi}), \qquad \text{(VII.48)}$$
$$\hat{z}_i = J_i(\hat{\psi}),$$

we are in a position to find the agreement $\hat{\psi}$. It must be noted however, that because of the complex structure of an agreement (see (VII.40)) there can be many agreements resulting in the same outcomes $(\hat{z}_o, \hat{z}_i)$.

VII.5 Concluding remarks

A model of a regional economy system in which a specific tax system is used has been formulated in this chapter. The economic policy in a region is based on the center-economic agents agreements, which means that the center controls the systems and at the same it takes advantage of the benefits bound up with cooperation.

The situation has been modelled as a dynamic game with the economic center announcing its strategy and firms playing a Nash game at the lower level. The distinguishing feature is that the center can negotiate with a firm an agreement about the conditions of applying one of the preferential tax formulas. We assume that such an agreement is not absolutely binding and in order to prevent breaking it a center uses threats of retaliation.

There are some aspects of the game which are worth emphasizing but have not been discussed in this paper. Take the question of credibility of threats as an example (Luh, Zheng, Ho 1984; Stefański, Straszak, 1986) which will be discussed in the following chapter of this volume. Next, we assumed that the center always will stick to its strategy; it might happen however that it is bluffing (Ho, Olsder, 1981). The question of the way in which the order of making agreements with subsequent firms influences the final solution is also worth discussing.

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