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Volume II: Applications

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Option pricing with Levy process in a fuzzy framework

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Abstract

In the following paper we propose a method for option pricing with application of stochastic analysis in a fuzzy framework. The process modeling the underlying asset is a geometric Levy process. It describes upward and downward jumps in price. In a fuzzy framework some parameters of the financial instrument cannot be precisely described and therefore they are introduced to the model as fuzzy numbers. Application of fuzzy arithmetics and stochastic analysis enabled us to consider different sources of uncertainty, not only the stochastic one. To obtain European call option pricing formula the minimal entropy martingale measure and Levy characteristics are used.

Keywords: finance, financial mathematics, Levy processes, fuzzy sets, Monte Carlo simulation.

1 Introduction

Classical financial model of option pricing, introduced by Black and Scholes (see [4, 11]), assumes continuity of the price process. Black-Scholes model delivers completeness of the market and uniqueness of derivative pricing. In this model the stochastic process which describes the price of an underlying financial instrument S is geometric Brownian motion. Black-Scholes formula is used to price

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derivatives by arbitrage and it is given in analytic form (see [11]). However, it is well documented in the finance literature that the Black-Scholes model does not describe the real behavior of financial instruments sufficiently well. The returns of logarithms of S in the real market have leptokurtic and skewed to the left distribution (see [5]). The second empirical phenomena is the volatility smile feature, i.e. the fact that the implied volatility is a convex function of the stike price (see [2, 3]). One of purposes of this paper is to improve the pricing model. In the paper sum of Brownian motion, drift and a linear combination of two Poisson processes is applied as model for the logarithm of asset prices. Such a process belongs to the class of Levy processes and its discontinuity may model positive and negative jumps of the underlying asset caused by external shocks. The proposed process is an extension of the model with one possible direction of jumps, considered in [14]. In the paper two techniques are used. Firstly, stochastic analysis, especially the martingale theory is applied. In this part Levy characteristics (see [15]) play very important role. As equivalent martingale measure the minimal entropy martingale measure (MEMM) is used. This measure has many applications in option pricing theory and economy. Since MEMM is closely related to exponential utility maximization, it has an important meaning in the problem of portfolio optimization and it was considered by [8, 9]. Secondly, in our approach, fuzzy sets theory and fuzzy arithmetics are applied (see [16, 17]). Fluctuating financial market and the lack of detailed information cause that many parameters of the model cannot always be described in the precise sense. This uncertainty is taken into account and expert opinions or imprecise estimates are introduced to the model in the form of fuzzy numbers. Applying this approach, financial analysts can receive the tool to pick option price with an acceptable belief degree for the latter use (see [16]). Computation techniques will be mainly applied to European call options. However, the proposed pricing method can be also used to other derivatives.

The paper is organized as follows. Section 2 contains preliminaries from stochastic analysis and fuzzy arithmetics. Section 3 presents basic definitions and facts concerning the minimal entropy martingale measure. In Section 4 the model of underlying asset is proposed and the pricing formula is proved. Section 5 is devoted to pricing the derivative with fuzzy parameters. Section 6 describes possibility of application of Monte Carlo simulations. The last section contains concluding remarks.

2 Notation and definitions

2.1 Stochastic preliminaries

We recall some basic facts about stochastic analysis and martingale method.

Let $\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, P\right)$ be a probability space with filtration satisfying standard assumptions. Let $T < \infty$.

A stochastic process $H = (H_t)_{t\in[0,T]}$ is cadlag, if its trajectories are functions, which are right-continuous with left limits.

H is $(\mathcal{F}_t)$-adapted if H_t is $(\mathcal{F}_t)$-measurable for each $t \in [0,T]$.

A probability measure Q on $(\Omega, \mathcal{F})$ is absolutely continuous with respect to P $(Q << P)$ if for all $A \in \mathcal{F}$

$$P(A) = 0 \Rightarrow Q(A) = 0$$

and it is equivalent to P if P and Q have the same sets with zero measure.

Let $\mathcal{S}_t$ be an $(\mathcal{F}_t)$-adapted cadlag stochastic process describing the underlying asset. Let r denote a constant risk–free interest rate and

$$\mathcal{Z}_t = e^{-rt}\mathcal{S}_t \tag{1}$$

be the discounted process of values of the underlying asset. We have to find the measure Q equivalent to P for which $\mathcal{Z}_t$ is a martingale.

The next step is to find the form of the process $\mathcal{S}_t$ with respect to this new probability measure Q. The price of a derivative with a payment function f is given by formula:

$$C_t = \exp\left(-r(T-t)\right)\mathbb{E}^Q\left(f\left(S\right)|\mathcal{F}_t\right), t \in [0,T]. \tag{2}$$

2.2 Fuzzy sets preliminaries

Now we remind some facts about fuzzy sets and numbers.

Let X be a universal set and $\tilde{A}$ be a fuzzy subset of X. We denote by $\mu_{\tilde{A}}$ its membership function $\mu_{\tilde{A}} : X \to [0,1]$, and by $\tilde{A}_\alpha = \{x : \mu_{\tilde{A}} \geq \alpha\}$ the α-level set of $\tilde{A}$, where $\tilde{A}_0$ is the closure of the set $\{x : \mu_{\tilde{A}} \neq 0\}$. In our paper we assume that $X = \mathbb{R}$.

Let $\tilde{a}$ be a fuzzy number. Then, under our assumptions, the α-level set $\tilde{a}_\alpha$ is a closed interval, which can be denoted by $\tilde{a}_\alpha = [\tilde{a}^L_\alpha, \tilde{a}^U_\alpha]$ (see e.g. [17]).

We can now introduce the arithmetics of any two fuzzy numbers. Let $\odot$ be a binary operator $\oplus$, $\ominus$, $\otimes$ or $\oslash$ between fuzzy numbers $\tilde{a}$ and $\tilde{b}$, where the binary operators correspond to $\circ$: $+$, $-$, $\times$ or $/$, according to the "Extension Principle" in [17]. Then the membership function of $\tilde{a} \odot \tilde{b}$ is defined by

$$\mu_{\tilde{a}\odot\tilde{b}}(z) = \sup_{(x,y):x\circ y=z} \min\{\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)\} \,. \tag{3}$$

Let $\odot_{\text{int}}$ be a binary operator $\oplus_{\text{int}}$, $\ominus_{\text{int}}$, $\otimes_{\text{int}}$ or $\oslash_{\text{int}}$ between two closed intervals $[a, b]$ and $[c, d]$. Then

$$[a, b] \odot_{\text{int}} [c, d] = \{z \in \mathbb{R} : z = x \circ y, \forall x \in [a, b], \forall y \in [c, d]\} \,, \tag{4}$$

where $\circ$ is an usual operation $+$, $-$, $\times$ and $/$, if the interval $[c, d]$ does not contain zero in the last case.

Therefore, if $\tilde{a}, \tilde{b}$ are fuzzy numbers, then $\tilde{a} \odot \tilde{b}$ is also the fuzzy number and its α-level set is given by

$$(\tilde{a} \oplus \tilde{b})_\alpha = \tilde{a}_\alpha \oplus_{\text{int}} \tilde{b}_\alpha = [\tilde{a}^L_\alpha + \tilde{b}^L_\alpha, \tilde{a}^U_\alpha + \tilde{b}^U_\alpha] \,,$$
$$(\tilde{a} \ominus \tilde{b})_\alpha = \tilde{a}_\alpha \ominus_{\text{int}} \tilde{b}_\alpha = [\tilde{a}^L_\alpha - \tilde{b}^U_\alpha, \tilde{a}^U_\alpha - \tilde{b}^L_\alpha] \,,$$
$$(\tilde{a} \otimes \tilde{b})_\alpha = \tilde{a}_\alpha \otimes_{\text{int}} \tilde{b}_\alpha =$$
$$= [\min\{\tilde{a}^L_\alpha\tilde{b}^L_\alpha, \tilde{a}^L_\alpha\tilde{b}^U_\alpha, \tilde{a}^U_\alpha\tilde{b}^L_\alpha, \tilde{a}^U_\alpha\tilde{b}^U_\alpha\}, \max\{\tilde{a}^L_\alpha\tilde{b}^L_\alpha, \tilde{a}^L_\alpha\tilde{b}^U_\alpha, \tilde{a}^U_\alpha\tilde{b}^L_\alpha, \tilde{a}^U_\alpha\tilde{b}^U_\alpha\}] \,,$$
$$(\tilde{a} \oslash \tilde{b})_\alpha = \tilde{a}_\alpha \oslash_{\text{int}} \tilde{b}_\alpha =$$
$$= [\min\{\tilde{a}^L_\alpha/\tilde{b}^L_\alpha, \tilde{a}^L_\alpha/\tilde{b}^U_\alpha, \tilde{a}^U_\alpha/\tilde{b}^L_\alpha, \tilde{a}^U_\alpha/\tilde{b}^U_\alpha\}, \max\{\tilde{a}^L_\alpha/\tilde{b}^L_\alpha, \tilde{a}^L_\alpha/\tilde{b}^U_\alpha, \tilde{a}^U_\alpha/\tilde{b}^L_\alpha, \tilde{a}^U_\alpha/\tilde{b}^U_\alpha\}] \,,$$

if α-level set $\tilde{b}_\alpha$ does not contain zero for all $\alpha \in [0, 1]$ in the case of $\oslash$.

Proposition 1. *Let $f : \mathbb{R} \to \mathbb{R}$ be a function, $\mathcal{F}(\mathbb{R})$ be a set of all fuzzy subsets of $\mathbb{R}$ and $\tilde{\Lambda} \in \mathcal{F}(\mathbb{R})$. We assume that the membership function $\mu_{\tilde{\Lambda}}$ of $\tilde{\Lambda}$ is upper semicontinuous and for all y the set $\{x : f(x) = y\}$ is compact. The function $f(x)$ can induce a fuzzy-valued function $\tilde{f} : \mathcal{F}(\mathbb{R}) \to \mathcal{F}(\mathbb{R})$ via the extension principle. Then the α-level set of $\tilde{f}(\tilde{\Lambda})$ is $\tilde{f}(\tilde{\Lambda})_\alpha = \{f(x) : x \in \tilde{\Lambda}_\alpha\}$.*

For proof of this proposition see [16].

Triangular fuzzy number $\tilde{a}$ with membership function $\mu_{\tilde{a}}(x)$ is defined as

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & \text{for } a_1 \le x \le a_2 \\ \frac{x-a_3}{a_2-a_3} & \text{for } a_2 \le x \le a_3 \\ 0 & \text{otherwise} \end{cases} \,, \tag{5}$$

where $[a_1, a_3]$ is the supporting interval and the membership function has peak in a_2. Triangular fuzzy number $\tilde{a}$ is denoted as

$$\tilde{a} = (a_1, a_2, a_3) \,.$$

Left-Right (or L-R) fuzzy numbers are special case of triangular fuzzy numbers (e.g. see [1, 6]), where linear functions used in the definition are replaced by monotonic functions, i.e.

Definition 1. *A fuzzy set $\tilde{A}$ on the set of real numbers is called L-R number if the membership function may be calculated as*

$$\mu_{\tilde{a}}(x) = \begin{cases} L\left(\frac{a_2-x}{a_2-a_1}\right) & \text{for } a_1 \le x \le a_2 \\ R\left(\frac{x-a_2}{a_3-a_2}\right) & \text{for } a_2 \le x \le a_3 \\ 0 & \text{otherwise} \end{cases}, \tag{6}$$

where L and R are continuous strictly decreasing function defined on $[0,1]$ with values in $[0,1]$ satisfying the conditions

$$L(x) = R(x) = 1 \text{ if } x = 0\,,\ L(x) = R(x) = 0 \text{ if } x = 1\,.$$

The L-R fuzzy number $\tilde{a}$ is denoted as

$$\tilde{a} = (a_1, a_2, a_3)_{LR}\,.$$

3 MEMM for Levy processes

The cadlag stochastically continuous process $Y = (Y_t)_{t\in[0,T]}$, $Y_0 = 0$ a.s., is called a Levy process if it satisfies the following conditions.

1. $Y_t - Y_s$ is independent of $\mathcal{F}_s$ for all $0 \le s \le t \le T$.
2. $Y_t - Y_s$ and Y_{t-s} have the same distributions for all $0 \le s \le t \le T$.

We assume that a truncation function φ is defined by the formula $\varphi(x) = xI_{|x|\le 1}$. We denote by $\mathcal{M}(\mathbb{R})$ the space of non-negative measures on $\mathbb{R}$. For Levy processes local characteristics, called Levy characteristics are defined. They are functions of the following form

$$B_t : [0,T] \to \mathbb{R},\, B_t = bt,$$

$$C_t : [0,T] \to \mathbb{R}, \quad C_t = ct,$$

$$\nu_t : [0,T] \to \mathcal{M}(\mathbb{R}), \quad \nu_t\,(dx) = \nu\,(dx)\,t,$$

$$\nu\,(\{0\}) = 0,\ \int_{\mathbb{R}} \left(|x|^2 \wedge 1\right) \nu\,(dx)$$

where $b, c \in \mathbb{R}$ and $\nu \in \mathcal{M}(\mathbb{R})$. Moreover, only constant b depends on the form of φ.

A stochastic process $S = (S_t)_{t\in[0,T]}$ is called geometric Levy process, if it can be written in the following form

$$S_t = S_0 \exp(Y_t), \quad t \in [0,T] \tag{7}$$

where Y_t is a Levy process.

Throughout this paper we assume that S is of the form (7), $\mathcal{F} = \mathcal{F}_T$ and for $t \in [0,T]$

$$\mathcal{F}_t = \sigma(S_s, s \in [0,t]) = \sigma(Y_s, s \in [0,t]).$$

The relative entropy $I(Q,P)$ of Q with respect to P is defined by

$$I(Q,P) = \begin{cases} E_P\left(\frac{dQ}{dP}\ln\left(\frac{dQ}{dP}\right)\right) & \text{if } Q << P \\ +\infty & \text{otherwise.} \end{cases}$$

If an equivalent martingale measure P_0 satisfies the inequality

$$I(P_0,P) \le I(Q,P)$$

for all equivalent martingale measures Q is called the minimal entropy martingale measure (MEMM).

Let

$$g^{(MEMM)}(\theta) = b + \left(\frac{1}{2}+\theta\right)c + \int_{\{|x|\le 1\}}\left((e^x-1)\,e^{\theta(e^x-1)} - x\right)\nu(dx)$$
$$+ \int_{\{|x|>1\}}(e^x-1)\,e^{\theta(e^x-1)}\nu(dx)$$

The following theorem from [12] will be useful in our paper.

Theorem 1. *If the equation*

$$g^{(MEMM)}(\theta) = r \tag{8}$$

has a solution θ_0, then the MEMM of S, $P_0 = P^{(MEMM)}$ exists. The process Y is also a Levy process under P_0 and the generating triplet of Y under P_0, (b_0, c_0, ν_0) has a form

$$b_0 = b + \theta_0 c + \int_{\{|x|\le 1\}} x\left(e^{\theta_0(e^x-1)} - 1\right)\nu(dx),$$
$$c_0 = c,$$
$$\nu_0(dx) = e^{\theta_0(e^x-1)}\nu(dx).$$

4 Pricing with crisp parameters

Let $\left(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, P\right)$ be a probability space with filtration. Let $T < \infty$.

The price of the underlying asset S_t is the geometric Levy process given by (7), where

$$Y_t = \mu t + \sigma W_t + k_1 N_t^{\kappa_1} + k_2 N_t^{\kappa_2}, \tag{9}$$

W_t is Brownian motion, $\sigma > 0$, $\mu, k \in \mathbb{R}$, $N_t^{\kappa_1}$ and $N_t^{\kappa_2}$ are Poisson processes with the intensities $\kappa_1 > 0$ and $\kappa_2 > 0$. We assume that W_t, $N_t^{\kappa_1}$ and $N_t^{\kappa_2}$ are independent. We usually also assume that $k_1 > 0$ and $k_2 < 0$, which describes upward and downward jumps in the underlying asset price.

Remark 1. *The process* (9) *can be also described as the compound Poisson process of the form*

$$Y_t = \mu t + \sigma W_t + \sum_{i=1}^{N_t^{\kappa}} \xi_i, \tag{10}$$

where W_t is also Brownian motion, $\sigma > 0$, $\mu, k \in \mathbb{R}$, N_t^{κ} is Poisson process with the intensity $\kappa = \kappa_1 + \kappa_2$ and $\{\xi_i\}_{i=1,2,\ldots}$ are random variables taking value k_1 with probability $p_1 = \frac{\kappa_1}{\kappa}$ and value k_2 with probability $p_2 = \frac{\kappa_2}{\kappa}$. Moreover, W_t, N_t^{κ} and $\xi_i,\ i = 1, 2, \ldots$ are independent. From probabilistic point of view they can be treated as the same processes, since they have the same finite dimensional distributions.

Theorem 2. *The price of European call option with strike price K and payment function $f(x) = (x - K)_+$ at time 0 is given by formula*

$$C_0 = e^{-\left(\kappa_1' + \kappa_2'\right)T} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\left(\kappa_1'\right)^m}{m!} \frac{\left(\kappa_2'\right)^n}{n!} T^{m+n} I_{m,n},$$

where

$$I_{m,n} = S_0 e^{(\mu_1 - r)T + \frac{\sigma^2 T}{2} + k_1 m + k_2 n} \Phi\left(d_+^{m,n}\right) - e^{-rT} K \Phi\left(d_-^{m,n}\right),$$

Φ is cumulative distribution function of standard normal distribution and θ_0 is the solution of the equation

$$\mu + \left(\frac{1}{2} + \theta\right)\sigma^2 + \kappa_1\left(e^{k_1} - 1\right) e^{\theta\left(e^{k_1} - 1\right)} + \kappa_2\left(e^{k_2} - 1\right) e^{\theta\left(e^{k_2} - 1\right)} = r, \tag{11}$$

$$\mu_1 = \mu + \theta_0 \sigma^2,\ \kappa_1' = \kappa_1 e^{\theta_0\left(e^{k_1} - 1\right)},\ \kappa_2' = \kappa_2 e^{\theta_0\left(e^{k_2} - 1\right)},$$

$$d_{-}^{m,n} = \frac{\ln \frac{S_0}{K} + \mu_1 T + k_1 m + k_2 n}{\sigma\sqrt{T}},$$
$$d_{+}^{m,n} = \frac{\ln \frac{S_0}{K} + \mu_1 T + \sigma^2 T + k_1 m + k_2 n}{\sigma\sqrt{T}}.$$

Proof. The proof is a modification of the proof of the corresponding theorem from [14]. We apply Theorem 1 to price the option. Equation (8) has the form (11). Since g is an increasing continuous function of θ $\lim_{\theta\to-\infty} g(\theta) = -\infty$ and $\lim_{\theta\to\infty} g(\theta) = \infty$, the above equation has an unique solution. We denote it by θ_0. According to Theorem 1, Y with respect to P_0 has the form

$$Y_t = \left(\mu + \theta_0\sigma^2\right) t + \sigma W_t^0 + k_1 N_t^{\kappa_1'} + k_2 N_t^{\kappa_2'}\,, t \in [0,T], \tag{12}$$

where W^0 is a Brownian motion, $N_t^{\kappa_1'}$ and $N_t^{\kappa_2'}$ are a Poisson processes with respect to P_0 and all the processes are independent. The price of the derivative is given by formula.

$$\begin{aligned}
C_0 &= e^{-rT}\mathbb{E}^{P_0}\left(S_T - K\right)^+ = e^{-rT}\mathbb{E}^{P_0}\left(S_T - K\right) I_{\{S_T>K\}} \\
&= \mathbb{E}^{P_0}\left(S_0 e^{(\mu_1-r)T+\sigma W_T^0+k_1 N_T^{\kappa_1'}+k_2 N_T^{\kappa_2'}} - e^{-rT}K\right) I_{\{S_T>K\}} \\
&= \mathbb{E}^{P_0}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} I_{\left\{N_T^{\kappa_1'}=m\right\}} I_{\left\{N_T^{\kappa_2'}=n\right\}} \Big(S_0 e^{(\mu_1-r)T+\sigma W_T^0+k_1 N_T^{\kappa_1'}+k_2 N_T^{\kappa_2'}} \\
&-e^{-rT}K\Big) I_{\left\{(\mu_1-r)T+\sigma W_T^0+k_1 N_T^{\kappa_1'}+k_2 N_T^{\kappa_2'}>\ln\frac{K}{S_0}-rT\right\}} \\
&= e^{-\left(\kappa_1'+\kappa_2'\right)T}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\left(\kappa_1'\right)^m}{m!}\frac{\left(\kappa_2'\right)^n}{n!}T^{m+n}\cdot \\
&\cdot\,\mathbb{E}^{P_0}\Big(S_0 e^{(\mu_1-r)T+\sigma W_T^0+k_1 N_T^{\kappa_1'}+k_2 N_T^{\kappa_2'}} \\
&-e^{-rT}K\Big) I_{\left\{(\mu_1-r)T+\sigma W_T^0+k_1 N_T^{\kappa_1'}+k_2 N_T^{\kappa_2'}>\ln\frac{K}{S_0}-rT\right\}}.
\end{aligned}$$

Since

$$(\mu_1 - r)\,T + \sigma W_T^0 + k_1 N_T^{\kappa_1'} + k_2 N_T^{\kappa_2'} \sim N\left(\left(\mu_1 - r\right)T + k_1 m + k_2 n, \sigma\sqrt{T}\right),$$

applying standard integral operations we obtain

$$\mathbb{E}^{P_0}\bigg(S_0 e^{(\mu_1-r)T+\sigma W_T^0+k_1 N_T^{\kappa_1'}+k_2 N_T^{\kappa_2'}}$$
$$-e^{-rT}K\big) I_{\left\{(\mu_1-r)T+\sigma W_T^0+k_1 N_T^{\kappa_1'}+k_2 N_T^{\kappa_2'}>\ln\frac{K}{S_0}-rT\right\}}$$
$$S_0 e^{(\mu_1-r)T+\frac{\sigma^2 T}{2}+k_1 m+k_2 n}\Phi\left(d_+^{m,n}\right)-e^{-rT}K\Phi\left(d_-^{m,n}\right)$$
$$=I_{m,n}.$$

□

Applying the Taylor expansion of the function exp, in further part of the paper, we will replace the pricing formula by

$$C_0=e^{-\left(\kappa_1'+\kappa_2'\right)T}\sum_{m=0}^{N}\sum_{n=0}^{N}\frac{\left(\kappa_1'\right)^m}{m!}\frac{\left(\kappa_2'\right)^n}{n!}T^{m+n}I_{m,n}, \tag{13}$$

for sufficiently large N.

5 Pricing with fuzzy parameters

We will indicate fuzzy parameters, writing the symbol ˜ above them. Other parameters will be treated as crisp numbers. One can find methodology similar to the one applied below in [16].

Let us assume that drift μ, volatility σ, interest rate r, k_1, k_2, κ_1 and κ_2 are not known precisely. Therefore we will treat them as L-R fuzzy numbers $\tilde{\mu},\tilde{\sigma},\tilde{r},\tilde{k}_1,\tilde{k}_2,\tilde{\kappa}_1$and $\tilde{\kappa}_2$. Let $\mu^*,\sigma^*,r^*,k_1^*,k_2^*,\kappa_1^*$ and κ_2^* be their defuzzified versions. We obtain the following form of the pricing formula (13)

$$\tilde{C}_0=e^{-\left(\tilde{\kappa}_1'\oplus\tilde{\kappa}_2'\right)\otimes T}\otimes\bigoplus_{m=0}^{N}\bigoplus_{n=0}^{N}\left(\left(\tilde{\kappa}_1'\right)^n\oslash m!\right)\otimes\left(\left(\tilde{\kappa}_2'\right)^n\oslash n!\right)\otimes T^{m+n}$$
$$\otimes\bigg(S_0\otimes e^{\left[((\tilde{\mu}'\ominus\tilde{r})\otimes T)\oplus(\tilde{\sigma}\otimes\tilde{\sigma}\otimes T\oslash 2)\oplus\tilde{k}_{m,n}\right]}$$
$$\otimes\tilde{\Phi}\left(\tilde{d}_+^{m,n}\right)\ominus\left(e^{-\tilde{r}\otimes T}\otimes K\otimes\tilde{\Phi}\left(\tilde{d}_-^{m,n}\right)\right)\bigg),$$

where $\tilde{k}_{m,n} = \left(\tilde{k}_1 \otimes m\right) \oplus \left(\tilde{k}_2 \otimes n\right)$,

$$\tilde{\mu}' = \tilde{\mu} \oplus (\theta_0 \otimes \tilde{\sigma} \otimes \tilde{\sigma}), \;\; \tilde{\kappa}'_{1,2} = \tilde{\kappa}_{1,2} \otimes e^{\theta_0 \otimes \left(e^{\tilde{k}_{1,2}} \ominus 1\right)},$$
$$\tilde{d}_{-}^{m,n} = \left[\ln(S_0 \oslash K) \oplus \left(\tilde{\mu}' \otimes T\right) \oplus \tilde{k}_{m,n}\right] \oslash \left(\tilde{\sigma} \otimes \sqrt{T}\right),$$
$$\tilde{d}_{+}^{m,n} = \left[\ln(S_0 \oslash K) \oplus \left(\tilde{\mu}' \otimes T\right) \oplus (\tilde{\sigma} \otimes \tilde{\sigma} \otimes T) \oplus \tilde{k}_{m,n}\right] \oslash \left(\tilde{\sigma} \otimes \sqrt{T}\right)$$

and θ_0 is the solution (with respect to θ) of the equation

$$\mu^* + (\sigma^*)^2 \left(\frac{1}{2} + \theta\right) + \kappa_1^* e^{\theta\left(e^{k_1^*} - 1\right)} \left(e^{k_1^*} - 1\right) + \kappa_2^* e^{\theta\left(e^{k_2^*} - 1\right)} \left(e^{k_2^*} - 1\right) = r^*. \tag{14}$$

In (14) the parameters with $*$ are defuzzified using one of the maximum methods (e.g. Mean of Maximum Method) for L-R numbers. The existence of solution for (14) follows from the same argument as in the proof of Theorem 2.

Applying Proposition 1, we can calculate the α-level set of $(\tilde{C}_0)_\alpha = \left[\tilde{C}_{0\alpha}^L, \tilde{C}_{0\alpha}^U\right]$, using corresponding combinations of $\tilde{r}_\alpha^L$ or $\tilde{r}_\alpha^U$, $\tilde{\mu}_\alpha^L$ or $\tilde{\mu}_\alpha^U$, $\tilde{\sigma}_\alpha^L$ or $\tilde{\sigma}_\alpha^U$, $\tilde{k}_{1,2\alpha}^L$ or $\tilde{k}_{1,2\alpha}^U$ and $\tilde{\kappa}_{1,2\alpha}^L$ or $\tilde{\kappa}_{1,2\alpha}^U$, respectively. From the Resolution identity

$$\mu_{\tilde{C}_0}(c) = \sup_{0 \le \alpha \le 1} \alpha I_{\left(\tilde{C}_0\right)_\alpha}(c)$$

we obtain the membership function of $\tilde{C}_0$. If c is the option price and $\mu_{\tilde{C}_0}(c) = \alpha$, then the value of the membership function may be treated by a financial analyst as the belief degree of c.

Remark 2. *We can extend the considered model of the underlying asset to the case of the linear combination of $K > 2$ independent Poisson processes, i.e.*

$$Y_t = \mu t + \sigma W_t + k_1 N_t^{\kappa_1} + k_2 N_t^{\kappa_2} + ... + k_K N_t^{\kappa_K}, \tag{15}$$

where W_t is Brownian motion, $\sigma > 0$, $\mu, k \in \mathbb{R}$, $N_t^{\kappa_1}$, $N_t^{\kappa_2}$,...,$N_t^{\kappa_K}$ are Poisson independent processes with the intensities $\kappa_1 > 0$ and $\kappa_2 > 0$,...,$\kappa_K > 0$. Then the pricing formula has the analogous form and Monte Carlo simulations can be applied. Moreover, this model can be equivalently treated as the model with the compound Poisson component

$$Y_t = \mu t + \sigma W_t + \sum_{i=1}^{N_t^\kappa} \xi_i, \tag{16}$$

where N_t^κ is Poisson process with the intensity $\kappa = \kappa_1 + \kappa_2 + ... + \kappa_K$ and independent random variables $\{\xi_i\}_{i=1,2,...}$ take value k_j with probability $p_j = \frac{\kappa_j}{\kappa}$, $j = 1, 2, \ldots K$.

6 Applications of Monte Carlo simulations

The fuzzy pricing formula for European call option may be numerically calculated for lower values of N. However, Monte Carlo simulations can be a better pricing method for more complicated derivatives.

In this section we discuss the possibility of application the simulation method to option pricing in the fuzzy framework. We conduct simulations in similar way as in [14].

We fix an α-level. For L-R fuzzy parameters $\tilde{\mu}, \tilde{\sigma}, \tilde{r}, \tilde{k}_1, \tilde{k}_2, \tilde{\kappa}_1, \tilde{\kappa}_2$,we find a solution θ_0 of (14) using their defuzzified counterparts $\mu^*, \sigma^*, r^*, k_1^*, k_2^*, \kappa_1^*$ and κ_2^*.

Simulations may be derived from the iterative version of process (12), which is analogous to formula known as Euler scheme in Black-Scholes model (see [10, 13]). In the considered case we have

$$S_{t_{i+1}} = S_{t_i} \exp \left(\left(\mu + \theta_0 \sigma^2 \right) \delta t + \sigma \sqrt{\delta t} \epsilon_i + k_1 \nu_{1i} + k_2 \nu_{2i} \right) , \qquad (17)$$

where $i = 0, 1, \ldots, m$ for $t_{i_0} = 0$ and $t_{i_m} = T$, $\delta t = t_{i+1} - t_i = \text{const}$, $\epsilon_0, \ldots \epsilon_{m-1}$ are *iid* random variables from standard normal distribution, $\nu_{10}, \ldots \nu_{1m-1}$ and $\nu_{20}, \ldots \nu_{2m-1}$ are *iid* realizations from Poisson processes with intensity κ_1' and κ_2' (compare with (12)). The value m is the number of steps in each trajectory.

We generate n trajectories via the formula (17), picking randomly value of each fuzzy parameter, treating the appropriate α-level set as an interval for uniform distribution. The derivative price is then computed as discounted (to the moment zero) mean value of payments from the considered financial instrument.

The Monte Carlo method can be also used instead of fuzzy pricing formula, obtained in Section 5.

7 Conclusions

In the paper a stochastic model for the underlying asset was introduced. The proposed process is a Levy process and it describes positive and negative jumps of the financial instrument prices. The combination of stochastic analysis and fuzzy theory enabled to obtain the pricing formula for European call option. Moreover, appropriate simulation techniques were described for the considered model. We also discussed a generalization of the obtained formulas for models with greater numbers of independent jumps.

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The papers presented in this Volume 2 constitute a collection of contributions, both of a foundational and applied type, by both well-known experts and young researchers in various fields of broadly perceived intelligent systems.
It may be viewed as a result of fruitful discussions held during the Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) organized in Warsaw on October 8, 2010 by the Systems Research Institute, Polish Academy of Sciences, in Warsaw, Poland, Institute of Biophysics and Biomedical Engineering, Bulgarian Academy of Sciences in Sofia, Bulgaria, and WIT - Warsaw School of Information Technology in Warsaw, Poland, and co-organized by: the Matej Bel University, Banska Bystrica, Slovakia, Universidad Publica de Navarra, Pamplona, Spain, Universidade de Tras-Os-Montes e Alto Douro, Vila Real, Portugal, and the University of Westminster, Harrow, UK:

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The consecutive International Workshops on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGNs) have been meant to provide a forum for the presentation of new results and for scientific discussion on new developments in foundations and applications of intuitionistic fuzzy sets and generalized nets pioneered by Professor Krassimir T. Atanassov. Other topics related to broadly perceived representation and processing of uncertain and imprecise information and intelligent systems have also been included. The Ninth International Workshop on Intuitionistic Fuzzy Sets and Generalized Nets (IWIFSGN-2010) is a continuation of this undertaking, and provides many new ideas and results in the areas concerned.

We hope that a collection of main contributions presented at the Workshop, completed with many papers by leading experts who have not been able to participate, will provide a source of much needed information on recent trends in the topics considered.

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