A ROBUST SEMI-MIXED 4-NODE SHELL ELEMENTS WITH ASSUMED ASYMMETRIC STRAINS AND STRESS RESULTANTS

S. Burzyński[1], J.Chróścielewski[1], K. Daszkiewicz[1] and W. Witkowski[1]

[1]*Faculty of Civil and Environmental Engineering, Gdańsk University of Technology,*
e-mail: kardaszk@pg.edu.pl

1. Introduction

4-node shell finite elements are commonly used in nonlinear analysis of shell structures. Standard displacement shell elements with fully integrated matrices are prone to shear and membrane locking and the problem of spurious zero energy modes appears with reduced integration. Alternatively, hybrid mixed finite elements based on multi-field variational principles may be used. The formulation of effective shell element plays crucial role in fast and accurate analysis of complex shell structures. It was shown in papers [1,2] that mixed elements developed from the 3-field Hu-Washizu functional allow for very large load steps in comparison to other elements. The present semi-mixed elements are developed in the framework of a nonlinear 6-parameter shell theory [3], where the reference surface is formally equivalent to the Cosserat surface. Hence, the measures of strains and resultant stresses are asymmetric. Some semi-mixed shell elements with asymmetric assumed stresses were proposed in [4], yet for different shell theory. While interpolation of asymmetric strains and enhanced strains was described in [5,6]. Recently, effective mixed shell element with asymmetric independent fields of strains and stress resultants were proposed in [7,8]. Here, the preliminary results for robust 3-field semi-mixed elements are presented based on [8].

2. Element formulation

The semi-mixed elements were developed based on modified 3-field Hu-Washizu functional. In the element formulation only membrane and shear components of strains and resultant stresses were treated as independent. The components of assumed stress resultants were interpolated in the following way

$$(1)\quad \bar{N}_{\mathrm{A}}^{11}=\alpha_1+\alpha_2\xi_2^*,\ \bar{N}_{\mathrm{A}}^{22}=\alpha_3+\alpha_4\xi_1^*,\ \bar{N}_{\mathrm{A}}^{12}=\alpha_5,\quad \bar{N}_{\mathrm{A}}^{21}=\alpha_6,\quad \bar{Q}_{\mathrm{A}}^{1}=\alpha_7+\alpha_8\xi_2^*,\ \bar{Q}_{\mathrm{A}}^{2}=\alpha_9+\alpha_{10}\xi_1^*,$$

$$(2)\quad \bar{N}_{\mathrm{B}}^{11}=\alpha_1+\alpha_2\xi_2^*,\ \bar{N}_{\mathrm{B}}^{22}=\alpha_3+\alpha_4\xi_1^*,\ \bar{N}_{\mathrm{B}}^{12}=\alpha_5+\alpha_6\xi_2^*,\ \bar{N}_{\mathrm{B}}^{21}=\alpha_7+\alpha_8\xi_1^*,\ \bar{Q}_{\mathrm{B}}^{1}=\alpha_9+\alpha_{10}\xi_2^*,\ \bar{Q}_{\mathrm{B}}^{2}=\alpha_{11}+\alpha_{12}\xi_1^*,$$

where $\xi_\alpha^*=\xi_\alpha-\bar{\xi}_\alpha$ are the so-called corrected natural coordinates, see [1]. Interpolation given by (1) was used in SMIX_A element, and by (2) in SMIX_B element. The first part of the strain field was interpolated in the same way as the stress field, while the second part according to EAS formulation, e.g. [6]. The ANS approach [9] was applied to transverse shear components of strains. The contravariant rule was used during transformation of resultant stresses and the first part of strains, while covariant rule for the second part of strains. The parameters for assumed stresses and strains were statically condensed at the element level.

3. Results

The proposed semi-mixed elements have correct rank and satisfy inf-sup condition and patch test. The performance of elements SMIX_A and SMIX_B was investigated by solving the well-known nonlinear test of pinched hemisphere with a hole. The geometry and material data are presented in Fig. 1a. Following [2] four times smaller shell thickness $h = 0.01$ was assumed to make example more prone to locking. The results for semi-mixed elements SMIX_A and SMIX_B were compared with the results for following 4-node shell elements: corresponding mixed elements MIX_A and MIX_B [7], enhanced strain element EANS4 [6] and semi-mixed element HW29 [2]. The computed nonlinear load-deflection curves are presented in Fig. 1b. The convergence rate is compared with the solutions obtained with alternative formulations in Table 1.

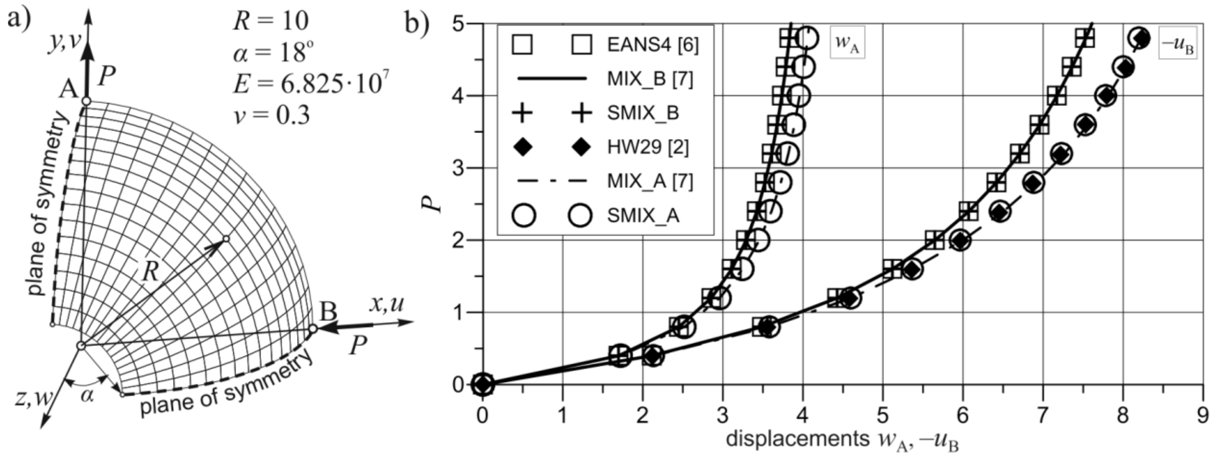

Figure 1: Pinched hemisphere with a hole, a) geometry, b) nonlinear equilibrium paths for 16×16 mesh.

Element	HW29	EANS4	MIX_A	MIX_B	SMIX_A	SMIX_B
Max Δ*P*	0.8	0.055	0.88	0.88	0.88	0.88
Total no. of iterations	61	518	30	38	33	36
CPU time [s]	-	856	32	40	28	31

Table 1: Comparison of maximum fixed load step Δ*P*, total number of iterations and process (CPU) time in nonlinear analysis for total load *P* = 8.8, 32×32 FE mesh (16×16 FE mesh for HW29).

4. Conclusions

The proposed semi-mixed shell elements require considerably less equilibrium iterations than elements EANS4 and HW29. The smaller number of independent parameters resulted in shorter CPU time than in the case of corresponding mixed elements [7]. The obtained equilibrium paths are in good agreement with the reference solutions. The element SMIX_B yield a slightly stiffer response than element SMIX_A.

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