STRUCTURAL OPTIMIZATION PROBLEM USING MICROTRUSS METHOD

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1. Abstract

Typically, the design of the structure is determined by conditions imposed on maximal stresses and characteristic displacement. However, in case of larger structures, the amount and cost of applied material becomes more important. Thus, it is necessary to optimize structure topology in order to obtain fully stressed design of minimal weight. The examples of optimal designs are *Michell's structures* such as structure transferring vertical load into two pinned supports, schematically presented in Figure 1. However, *the structures* are characterized by infinitely dense distribution of members and thus they are not directly applicable in engineering practice. On the other hand, the optimal layouts can be obtained by relaxation of the problem of two-material design with the use of homogenization methods. However, the obtained designs are often characterized by checkerboard phenomenon, which significantly hampers their manufacturing and practical realization. The purpose of this research is to propose a novel method for optimal design of skeletal structures, which is deprived of the above mentioned drawbacks. The introduced method provides forming the optimum morphology by discretizing the design domain with micro-trusses and by gradual modification of the layout in order to obtain minimal weight design.

2. Optimal structural form of Michell

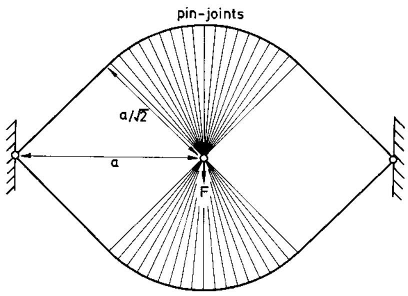


Figure 1: Approximation of Michell's truss by structure of finite number of members

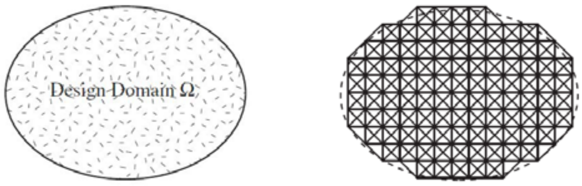
We consider the inverse analytical problem to create an efficient structural form which has given the design conditions of materials, loads and boundary. It had been suggested that the optimum structure of this problem bases on the principle stress line on his theoretic approach (Figure 1). This classical problem is paid attention recently to create the digital design as the model of bench mark in the field of computing mechanics. It is well-known that this problem is as the Michell's optimum truss [1].

3. OPTIMISATION STRATEGY [2]

3.1. Optimum structural form using microtruss

We represent the design domain Ω of a continuum by micro-truss $\Delta\Omega^{(m)}$, (1). The discretisation is diagrammatically represented in Figure 2, where an example of a continuum Figure 2(a) is discretised as Figure 2(b). It is proposed that the behaviour of the domain Ω can be adequately modelled by microtrusses for large number of unit cells, M, where each microtruss member represents a pin-jointed linear extensional spring.

(1)
$$\Omega = \int \mathrm{d}\Omega = \lim_{\mathrm{d}\Omega\to 0} \lim_{n\to\infty} \sum_{m=1}^{n} \mathrm{d}\Omega^{(m)} \approx \sum_{m}^{M} \Delta\Omega^{(m)}$$



(a) Continuum, Ω (b) FEM of microtruss, $\sum \Delta\Omega$

Figure 2: Discretisation of a Continuum

3.2. Method of optimal morphogenesis

The design variable is the cross-sectional area of the microtruss members, $\boldsymbol{x} \in \boldsymbol{R}^N$ of $\Delta\Omega$ and therefore the linear equilibrium equation can be rewritten as (2).

$$\boldsymbol{F}(\boldsymbol{u}, f, \boldsymbol{p}, \boldsymbol{x}) = K(\boldsymbol{x})\boldsymbol{u} - f\boldsymbol{p} = \boldsymbol{0} \tag{2}$$

where $\boldsymbol{u} \in \boldsymbol{R}^N$ is a displacement vector, $f \in \boldsymbol{R}$ is a load parameter and $\boldsymbol{p} \in \boldsymbol{R}^N$ is a load vector.

The goal of optimization is a minimum weight design, represented as (3) in a discrete form. Since the density and the member length are constant, a more optimum form can be formed by changing only the cross-sectional area of each member.

$$\begin{aligned} &\text{Minimize}: \sum_{m=1}^{M} W^{(m)} = \sum_{m=1}^{M} \rho A_{(\nu)}^{(m)} \ell^{(m)} \\ &\text{subject to}: \boldsymbol{F}(\boldsymbol{u}, f, \boldsymbol{p}, \boldsymbol{x}) = \boldsymbol{0} \\ &\qquad A_{(\nu)}^{(m)} \leq A_{max}, \quad \sigma_{(\nu)}^{(m)} \leq \sigma_a \end{aligned} \tag{3}$$

where $W^{(m)}$ is the weight of each member, ρ is the density, $\ell^{(m)}$ is the member length, A_{max} is the maximum cross-sectional area and σ_a is the allowable stress. The design modification is based on the ratio of the member stress and the average stress $\overline{\sigma}_{(\nu)}$, where the stiffness of a member is updated at each load step according to (4).

$$\boldsymbol{x}_{(\nu+1)} = \mathcal{F}(\boldsymbol{x}_{(\nu)}) = \gamma \left(\frac{\sigma_{(\nu)}^{(m)}}{\overline{\sigma}_{(\nu)}} \right)^2 \boldsymbol{x}_{(\nu)} \tag{4}$$

where $\overline{\sigma}_{(\nu)} = \frac{1}{M}\sqrt{\sum_{m=1}^{M} \left(\sigma_{(\nu)}^{(m)}\right)^2}, \quad \nu = 1, 2, \cdots, \quad \gamma$ is optimization rate constant.

4. CONCLUDING REMARKS

In this research, we have constructed a presented optimization method using microtruss method and addressed the problem of structure optimization using coat hanging problem and simple beam model as an example. It is the greatest result of this presented method that the layout of the skeleton structure with the minimum weight is obtained by placing the microtruss in the design area and applying the load and the boundary condition.

References

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[2] I. Ario, M. Nakazawa, Y. Tanaka, I. Tanikura, S. Ono. Development of a prototype deployable bridge based on origami skill, Automation in Construction, Vol.32, 104-111, 2013.