APPLICATION OF EQUILIBRIUM-BASED FINITE ELEMENT METHOD IN TOPOLOGY OPTIMIZATION PROBLEMS

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1. Introduction

The displacement-based finite element method (FEM) is usually utilized as a computational tool for solving topology optimization problems. Such an approach has been described e.g. in [2,1,3]. In the present work, the equilibrium problem has been solved by use of the stress-based FEM which follows from the principle of complementary work or – alternatively – from the principle of minimum of the complementary energy functional. As mentioned in [2], the topology optimization problem can be formulated as the following compliance minimization problem:

$$\min_{D\in\mathsf{E}_{\mathrm{ad}}} \min_{\boldsymbol{\tau}\in Y_{b,t}} \left\{ \frac{1}{2}\, b(\boldsymbol{\tau},\boldsymbol{\tau}) \right\} \qquad \text{with} \quad b(\boldsymbol{\sigma},\boldsymbol{\tau}) = \int_\Omega C_{ijkl}\,\sigma_{ij}\,\tau_{kl}\,\mathrm{d}x \tag{1}$$

where D_{ad} denotes a set of admissible elasticity tensors D_{ijkl}, $C_{ijkl} \equiv D^{-1}_{ijkl}$, and $Y_{b,t}$ is the set of statically admissible fields of stress tensors

$$Y_{b,t} = \left\{ \tau_{ij} \in L^2(\Omega) : \tau_{ij} = \tau_{ji},\ \tau_{ji,j} + b_i = 0 \text{ in } \Omega,\ \tau_{ji}\,n_j = t_i \text{ on } \Gamma_\sigma \right\}. \tag{2}$$

Let us consider continuous distribution of the material described by a density function $\varrho(\boldsymbol{x})$, so the elasticity and compliance tensors can be expressed as follows (SIMP method, e.g. [2,1]):

$$D_{ijkl}(\boldsymbol{x}) = [\varrho(\boldsymbol{x})]^p\, D^0_{ijkl}, \quad C_{ijkl}(\boldsymbol{x}) = [\varrho(\boldsymbol{x})]^{-p}\, C^0_{ijkl}, \quad C^0_{ijkl} = (D^0_{ijkl})^{-1} \qquad \text{with } p > 3. \tag{3}$$

After using Eq. (3), the problem (1) can be written as the minimization problem:

$$\min_{\varrho(\boldsymbol{x})} b(\boldsymbol{\sigma},\boldsymbol{\sigma}) \tag{4}$$

provided that

$$b(\boldsymbol{\sigma},\boldsymbol{\tau}) = 0 \quad \forall \boldsymbol{\tau} \in Y_{0,0}, \quad \boldsymbol{\sigma} \in Y_{b,t}, \tag{5}$$

$$C_{ijkl}(\boldsymbol{x}) = [\varrho(\boldsymbol{x})]^{-p}\, C^0_{ijkl}, \tag{6}$$

$$\int_\Omega \varrho(\boldsymbol{x})\,\mathrm{d}x \le V, \qquad 0 < \varrho_{\min} \le \varrho \le 1 \tag{7}$$

where $Y_{0,0} = Y_{b,t}|_{b_i=0,\,t_i=0}$ and V is the given volume of the structure.

2. Stress-based finite element solution

The plane stress problem has been analyzed in the present work. To solve the equation of complementary work (5), the stress fields satisfying the equilibrium equations inside domain Ω have been constructed by means of the Airy stress function which has been interpolated with the help of rectangular hermitian element with 16 degrees of freedom. The equilibrium conditions given on the boundary of the design domain Γ_σ have a form of linear constraints and have been satisfied by use of the Lagrange multiplier method. To make the application of this approach easier, additional elements have been implemented on edges of element located on Γ_σ. The detailed description of the stress-based approach can be found in [4].

3. Example

A rectangular design domain with ratio 6:1 has been considered. The vertical load distributed locally along a short segment has been assumed in the middle of the upper edge of the design region. The structure is considered to be simply supported at the central points of the vertical edges of the region.

The optimum solution has been found by use of two rectangular element grids: 150×50 and 300×100 elements to dicretize the right half of the design area. The same numbers of rectangular elements with 8 degrees of freedom have been utilized to find the solution based on the displacements method in order to compare the proposed approach with the well known method described in [1].

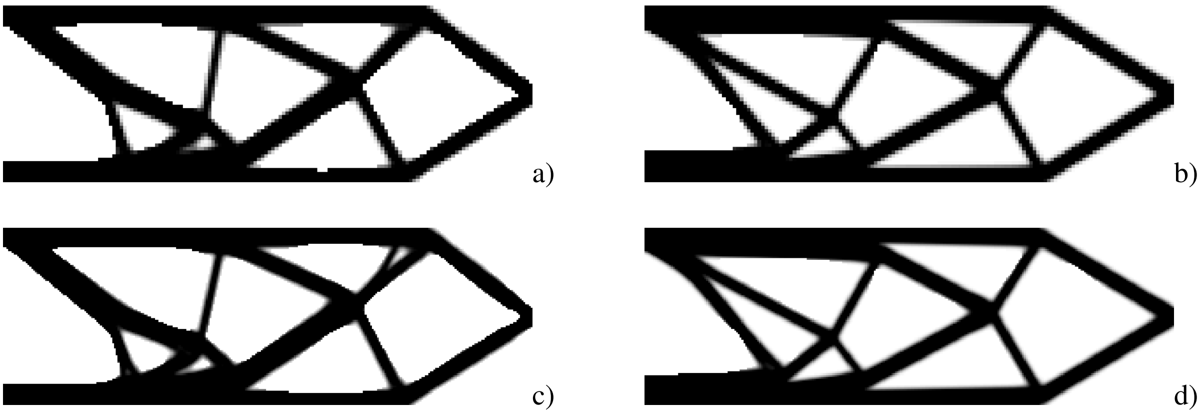

Figure 1: Optimized solution obtained by: a) stress-based approach, 150×50 elements; b) displacement-based approach, 150×50 elements; c) stress-based approach, 300×100 elements; d) displacement-based approach, 300×100 elements. Calculations made with volume fraction $V = 0.4$.

Although both the solutions, the stress and displacement ones, look similar, some differences related to the nature of the two approaches can be noticed. The image of the stress-based solution seems to be sharper a little than that obtained by the displacement approach. The stress-based approach has appeared to be more time consuming that the displacement-based one as expected. However, the proposed method has required much smaller number of iterations to satisfy the assumed tolerance.

Mesh resolution	Number of iterations		Execution time [s]	
	Stress method	Displacement method	Stress method	Displacement method
150×50	64	145	209	23.1
300×100	67	325	1409	224

Table 1: Comparison of efficiency of two applied methods.

References

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