LI

ON A METHOD PROPOSED BY PROFESSOR BADANO FOR THE SOLUTION OF ALGEBRAIC EQUATIONS*

Communicated 4 August 1842.

[*Proc. Roy. Irish Acad.* vol. II (1844), pp. 275–6.]

The President made a communication respecting a method which had been lately proposed by Professor Badano of Genoa, for the solution of algebraical equations of the fifth and higher degrees.†

Lagrange has shown that the function

$$t^5 = (x' + \omega x'' + \omega^2 x''' + \omega^3 x^{\text{IV}} + \omega^4 x^{\text{V}})^5$$

receives only twenty-four different values, for all possible changes of arrangement of the five quantities, $x', \ldots, x^{\text{V}}$, if ω be an imaginary root of unity, so that

$$\omega^4 + \omega^3 + \omega^2 + \omega + 1 = 0.$$

Professor Badano has proposed to express these twenty-four values by certain combinations of quadratic and cubic radicals, suggested by the theory of biquadratic equations, and having the following for their type:

$$t^5 = H_1 + \sqrt{H_2} + \sqrt[3]{H_3 + \sqrt{H_4}} + \sqrt[3]{H_5 - \sqrt{H_6}} + \sqrt{\{H_7 + \sqrt{H_8} + \sqrt[3]{H_9 + \sqrt{H_{10}}} + \sqrt[3]{H_{11} - \sqrt{H_{12}}}\}}$$
$$+ \sqrt{\{H_{13} + \sqrt{H_{14}} + \theta\sqrt[3]{H_{15} + \sqrt{H_{16}}} + \theta^2\sqrt[3]{H_{17} - \sqrt{H_{18}}}\}}$$
$$+ \sqrt{\{H_{19} + \sqrt{H_{20}} + \theta^2\sqrt[3]{H_{21} + \sqrt{H_{22}}} + \theta\sqrt[3]{H_{23} - \sqrt{H_{24}}}\}};$$

θ being here an imaginary cube-root of unity. He contends that the twenty-four quantities, $H_1, \ldots, H_{24}$, are all symmetric functions of the five quantities $x', \ldots, x^{\text{V}}$; and that they are connected among themselves by the sixteen relations

$$H_3 = H_5, \quad H_4 = H_6, \quad H_7 = H_{13} = H_{19}, \quad H_8 = H_{14} = H_{20}, \quad H_9 = H_{15} = H_{21},$$
$$H_{10} = H_{16} = H_{22}, \quad H_{11} = H_{17} = H_{23}, \quad H_{12} = H_{18} = H_{24}, \quad H_9 = H_{11}, \quad H_{10} = H_{12}.$$

Sir W. Hamilton examines, in great detail, the composition of the two conjugate quantities H_4, H_6, which are each of the thirtieth dimension relatively to the five original quantities $x', \ldots, x^{\text{V}}$; and arrives at the conclusion that neither H_4 nor H_6 is a symmetric function of those five quantities $x', \ldots, x^{\text{V}}$, though each is symmetric relatively to four of them. He finds also that these two quantities H_4 and H_6 are not generally equal to each other, but differ by the sign of an imaginary radical, namely,

$$(\theta - \theta^2)(\omega - \omega^2 - \omega^3 + \omega^4) = \sqrt{-15},$$

* [This is an abstract of the next paper, LII.]

† *Nuove Ricerche sulla Risoluzione Generale delle Equazioni Algebriche* del P. Gerolamo Badano, Carmelitano scalzo, Professore di Matematica nella R. Universita di Genova. Genova, Tipografia Ponthenier, 1840. See also an 'Appendice' to the same work.

when they are fully developed, in consistency with Professor Badano's definitions. Analogous results are obtained for the two quantities H_3, H_5; and these general results are verified by applying them to a particular system of numerical values of the five quantities $x', \ldots, x^{V}$. It is shown also that the three quantities H_7, H_{13}, H_{19}, are neither independent of the arrangement of those five quantities x, nor (generally) equal to each other. And thus, although H_1 is symmetric, and H_2 vanishes, Sir W. H. conceives it to be proved that Professor Badano's expressions, for the twenty-four values of Lagrange's function t^5, give no assistance towards the solution of the general equation of the fifth degree, and therefore that the same method could not be expected to resolve equations still more elevated, even if we were not in possession of an *à priori* proof that no root of any general equation above the fourth degree can be expressed as a function of its coefficients, by any finite combination of radicals and rational functions.