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LANDEN.

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LANDEN, JOHN, a distinguished mathematician of the 18th century, was born at Peakirk near Peterborough in Northamptonshire in 1719, and died 15th January 1790 at Milton in the same county. Most of his time was spent in the pursuits of active life, but he early showed a strong talent for mathematical study, which he eagerly cultivated in his leisure hours. In 1762 he was appointed agent to the Earl Fitzwilliam, and held that office to within two years of his death. He lived a very retired life, and saw little or nothing of society; when he did mingle in it, his dogmatism and pugnacity caused him to be generally shunned. He was first known as a mathematician by his essays in the *Ladies' Diary* for 1744. In 1766 he was elected a Fellow of the Royal Society. He was well acquainted and *au courant* with the works of the mathematicians of his own time, and has been called the English D'Alembert. In his *Discourse* on the "Residual Analysis," in which he proposes to substitute for the method of fluxions a purely algebraical method, he says, "It is by means of the following theorem, viz.

$$\frac{x^{\frac{m}{n}} - v^{\frac{m}{n}}}{x - v} = x^{\frac{m}{n}-1} \times 1 + \frac{v}{x} + \left(\frac{v}{x}\right)^2 + \ldots (m \text{ terms})$$

$$\div 1 + \left(\frac{v}{x}\right)^{\frac{m}{n}} + \left(\frac{v}{x}\right)^{\frac{2m}{n}} + \ldots (n \text{ terms})$$

(where m and n are integers), that we are able to perform all the principal operations in our said analysis; and I am not a little surprised that a theorem so obvious, and of such vast use, should so long escape the notice of algebraists." The idea is of course a perfectly legitimate one, and may be compared with that of Lagrange's *Calcul des Fonctions*. His memoir (1775) on the rotatory motion of a body contains (as the author was aware) conclusions at variance with those arrived at by D'Alembert and

Euler in their researches on the same subject. He reproduces and further develops and defends his own views in his *Mathematical Memoirs,* and in his paper in the *Philosophical Transactions* for 1785. But Landen's capital discovery is that of the theorem known by his name (obtained in its complete form in the memoir of 1775, and reproduced in the first volume of the *Mathematical Memoirs*) for the expression of the arc of an hyperbola in terms of two elliptic arcs. To find this, he integrates a differential equation derived from the equation

$$t = gx\sqrt{\frac{m^2 - x^2}{m^2 - gx^2}},$$

interpreting geometrically in an ingenious and elegant manner three integrals which present themselves. If in the foregoing equation we write $m = 1$, $g = k^2$, and instead of t consider the new variable $y = t \div (1 - k')$, then

$$y = (1 + k')\, x\sqrt{\frac{1 - x^2}{1 - k^2x^2}},$$

which is the form known as Landen's transformation in the theory of elliptic functions; but his investigation does not lead him to obtain the equivalent of the resulting differential equation

$$\frac{dy}{\sqrt{1 - y^2 \,.\, 1 - \lambda^2 y^2}} = \frac{(1 + k')\, dx}{\sqrt{1 - x^2 \,.\, 1 - k^2x^2}}, \text{ where } \lambda = \frac{1 - k'}{1 + k'},$$

due it would appear to Legendre, and which (over and above Landen's own beautiful result) gives importance to the theorem as leading directly to the quadric transformation of an elliptic integral in regard to the modulus.

The list of his writings is as follows:—*Ladies' Diary,* various communications, 1744—1760; papers in the *Phil. Trans.,* 1754, 1760, 1768, 1771, 1775, 1777, 1785; *Mathematical Lucubrations,* 1755; *A Discourse concerning the Residual Analysis,* 1758; *The Residual Analysis,* book I., 1764; *Animadversions on Dr Stewart's Method of computing the Sun's Distance from the Earth,* 1771; *Mathematical Memoirs,* 1780, 1789.