959.

NOTE ON PLÜCKER'S EQUATIONS.

[From the *Messenger of Mathematics*, vol. XXIV. (1895), pp. 23, 24.]

It is well known that if

$$A,\ B,\ C,\ D = 2,\ 3,\ 6,\ 8,$$

then the equations

$$\begin{aligned} n &= \ m^2 - \ m - A\delta - B\kappa, \\ \iota &= 3m^2 - 6m - C\delta - D\kappa, \\ m &= \ n^2 - \ n - A\tau - B\iota, \\ \kappa &= 3n^2 - 6n - C\tau - D\iota, \end{aligned}$$

are equivalent to three independent equations giving n, τ, ι in terms of m, δ, κ. It is easy to show that the *necessary* conditions in order that this may be so, are

$$C = 3A, \quad \text{and} \quad D = 3B - 1,$$

that is,

$$A,\ B,\ C,\ D = A,\ B,\ 3A,\ 3B-1,$$

where A and B are arbitrary.

In fact, from the last two equations eliminating τ, and for n, ι substituting their values, we have

$$\begin{aligned} Cm - A\kappa = &\ (C-3A)(m^2 - m - A\delta - B\kappa)^2, \\ &- (C-6A)(m^2 - m - A\delta - B\kappa), \\ &+ (AD - BC)(3m^2 - 6m - C\delta - D\kappa), \end{aligned}$$

which must therefore be an identity. In order that the term in m^4 may vanish we must have $C = 3A$; and then substituting this value for C, we must have

$$3Am - A\kappa = 3A(m^2 - m - A\delta - B\kappa) + (AD - 3AD)(3m^2 - 6m - 3A\delta - D\kappa).$$

Here the coefficient of m^2 must vanish, that is,

$$0 = 3A + 3AD - 9AB, \quad \text{or} \quad D = 3B - 1,$$

and, substituting this value, the equation is

$$\begin{aligned} 3Am - A\kappa = &\ 3A(-m - A\delta - B\kappa) \\ &- A\{-6m - 3A\delta - (3B-1)\kappa\}, \end{aligned}$$

that is,

$$\begin{aligned} 3m - \kappa = &- 3m - 3A\delta - 3B\kappa \\ &+ 6m + 3A\delta + (3B-1)\kappa, \end{aligned}$$

an identity.