726.

A FORMULA BY GAUSS FOR THE CALCULATION OF LOG 2 AND CERTAIN OTHER LOGARITHMS.

[From the *Messenger of Mathematics,* vol. VIII. (1879), pp. 125, 126.]

GAUSS has given, *Werke*, t. II., p. 501, a formula which is in effect as follows:

$$2^{196} = 10^{59}\left(\frac{1025}{1024}\right)^5\left(\frac{1048576}{1048575}\right)^8\left(\frac{6560}{6561}\right)^3\left(\frac{15624}{15625}\right)^8\left(\frac{9801}{9800}\right)^4,$$

viz. this is

$$= 2^{59}.5^{59}\left(\frac{5^2.41}{2^{10}}\right)^5\left(\frac{2^{20}}{5^2.3.11.31.41}\right)^8\left(\frac{5.2^5.41}{3^8}\right)^3\left(\frac{2^3.3^2.7.31}{5^6}\right)^8\left(\frac{3^4.11^2}{2^3.5^2.7^2}\right)^4,$$

where on the right-hand side the several prime factors have the indices following, viz.

$$\begin{array}{lll} 2, & \text{index is} & (59+160+15+24-50-12)=196, \\ 3 & \quad ,, & (16+16-8-24)=0, \\ 5 & \quad ,, & (59+10+3-16-48-8)=0, \\ 7 & \quad ,, & (8-8)=0, \\ 11 & \quad ,, & (8-8)=0, \\ 31 & \quad ,, & (8-8)=0, \\ 41 & \quad ,, & (5+3-8)=0, \end{array}$$

or the right-hand side is $=2^{196}$ as it should be. The value of $\log 2$ calculated from $2^{196}=10^{59}$ is $\log 2 = \frac{59}{196} = \cdot 301020$, viz. there is an error of a unit in fifth place of decimals. The actual value of 2^{196} has been given me by Mr Glaisher:

$$2^{196} = 10043\ 36277\ 66186\ 89222\ 13726\ 30771$$
$$32266\ 26576\ 37687\ 11142\ 45522\ 06336.^*$$

Supposing log 2 calculated by the form, we then have

$$41 = (\tfrac{1025}{1024})\, 2^{12} \div 10^2, \text{ giving } \log 41,$$

and

$$3^8 = 10 . \tfrac{6561}{6560} . 2^4 . 41, \text{ giving } \log 3;$$

and formulæ may be obtained proper for the calculation of the logarithms of $\frac{11}{7}$, 11.31, and 7.31.

* The value was deduced by Mr Glaisher from Mr Shanks's value of 2^{193} in his *Rectification of the Circle*, (1853), p. 90.