908.

ON TWO INVARIANTS OF A QUADRIQUADRIC FUNCTION.

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THE quadriquadric function

$$\begin{aligned} &z^2\ (ax^2+2hxy+g'y^2)\\ &+2zw\ (h'x^2+2bxy+fy^2)\\ &+\ w^2\,(gx^2+2f'xy+cy^2),\end{aligned}$$

considered successively as a function of (z, w) and of (x, y), has the discriminants U, V, equal to

$$(ax^2+2hxy+g'y^2)(gx^2+2f'xy+cy^2)-(h'x^2+2bxy+fy^2)^2,$$

$$(az^2+2h'zw+gw^2)(g'z^2+2fzw+cw^2)-(hz^2+2bzw+f'w^2)^2,$$

respectively. As is well known, these quartic functions have each of them the same quadrinvariant and the same cubinvariant; these are the invariants in question of the quadriquadric function.

The quadrinvariant has been calculated in a different notation, but I am not aware that the cubinvariant has been before calculated; the two values are as follows:

Quadrinvariant is

a^2c^2	+ 3
ab^2c	− 24
b^4	+ 48
$acgg'$	+ 42
$achf'$	− 12
$ach'f$	− 12
$abff'$	+ 72
$bchh'$	+ 72
b^2gg'	− 24
b^2fh'	− 96
$b^2f'h$	− 96
af^2g	− 36
af'^2g'	− 36
$bfgh$	+ 72
$bf'g'h'$	+ 72
cgh^2	− 36
$cg'h'^2$	− 36
$g^2g'^2$	+ 3
$f^2h'^2$	+ 48
f'^2h^2	+ 48
$gg'hf'$	− 12
$gg'h'f$	− 12
$ff'hh'$	− 48
	±480

Cubinvariant is

a^3c^3	− 1
$a^2b^2c^2$	+12
ab^4c	−48
b^6	+64
	+76
	−49

a^2c^2gg'	+ 33
ab^2cgg'	−120
a^2c^2fh'	+ 6
$a^2c^2f'h$	+ 6
ab^2cfh'	+ 24
$ab^2cf'h$	+ 24
b^4fh'	−192
$b^4f'h$	−192
b^4gg'	− 48
a^2bcff'	− 36
abc^2hh'	− 36
ab^3ff'	+144
b^3chh'	+144
	+381
	−624

a^2cf^2g	− 36
$a^2cf'^2g'$	− 36
ac^2gh^2	− 36
$ac^2g'h'^2$	− 36
ab^2f^2g	− 72
$ab^2f'^2g'$	− 72
b^2cgh^2	− 72
$b^2cg'h'^2$	− 72
$abcfgh$	+180
$abcf'g'h'$	+180
b^3fgh	+144
$b^3f'g'h'$	+144
	+648
	−432

$acg^2g'^2$	+ 33
$acf'gg'h$	− 60
$acfgg'h'$	− 60
$acf^2h'^2$	+ 24
acf'^2h^2	+ 24
$acff'hh'$	+ 12
$abff'gg'$	+ 180
$abf^2f'h'$	− 144
$abff'^2h$	− 144
$bcgg'hh'$	+ 180
$bcfhh'^2$	− 144
$bcf'h^2h'$	− 144
$a^2f^2f'^2$	+ 54
$c^2h^2h'^2$	+ 54
$b^2f^2h'^2$	+ 192
$b^2f'^2h^2$	+ 192
$b^2ff'hh'$	+ 96
$b^2fgg'h'$	+ 24
$b^2f'gg'h$	+ 24
$b^2g^2g'^2$	+ 12
	+1101
	− 696

$af^2f'gh$	− 36
$aff'^2g'h'$	− 36
af^2g^2g'	− 36
$af'^2gg'^2$	− 36
af^3gh'	+ 72
$af'^3g'h$	+ 72
$cfgh^2h'$	− 36
$cf'g'hh'^2$	− 36
$cg^2g'h^2$	− 36
$cgg'^2h'^2$	− 36
$cf'gh^3$	+ 72
$cfg'h'^3$	+ 72
$bff'g'h'^2$	−144
$bf'^2g'hh'$	−144
bf^2ghh'	−144
$bff'gh^2$	−144
$bfg^2g'h$	− 36
$bf'gg'^2h'$	− 36
	+288
	−936

$g^3g'^3$	− 1
$fg^2g'^2h'$	+ 6
$f'g^2g'^2h$	+ 6
$f^2gg'h'^2$	+ 24
$f'^2gg'h^2$	+ 24
$ff'gg'hh'$	+ 12
$f^3h'^3$	− 64
f'^3h^3	− 64
$f^2g^2h^2$	+ 54
$f'^2g'^2h'^2$	+ 54
$f^2f'hh'^2$	+ 96
ff'^2h^2h'	+ 96
	+ 372
	− 129
	±2866

By writing herein $f', g', h' = f, g, h$, we obtain of course the two invariants of the symmetrical quadriquadric function.