# 698.

## ON A THEOREM RELATING TO COVARIANTS.

[From the *Journal für die reine und angewandte Mathematik* (Crelle), t. LXXXVII. (1878), pp. 82, 83.]

THE theorem given by Prof. Sylvester, *Crelle*, vol. LXXXV., p. 109, may be stated as follows: If for a binary quantic of the order $i$ in the variables, we consider the whole system of covariants of the degree $j$ in the coefficients, then

$$\Sigma\theta(k+1) = \frac{\Pi(i+j)}{\Pi(i)\,\Pi(j)},$$

where $\theta$ denotes the number of asyzygetic covariants of the order $\theta$ in the variables, the values of $\theta$ being $ij$, $ij-2$, $ij-4$, ..., 1 or 0, according as $ij$ is odd or even.

In the case of the binary quintic $(a, \ldots \between x, y)^5$, $(i=5)$, we have a series of verifications in the Table 88 of my "Ninth Memoir on Quantics," *Phil. Trans.* vol. CLXI. (1871), [462]: viz. writing the small letters $a, b, c, \ldots, u, v, w$ (instead of the capitals $A$, $B$, etc.) to denote the covariants of the quintic, $a$, the quintic itself, degree 1, order 5, or as I express it, deg-order 1.5: $b$, the covariant deg-order 2.2, etc., the whole series of deg-orders being

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $a$, | $b$, | $c$, | $d$, | $e$, | $f$, | $g$, | $h$, | $i$, | $j$, | $k$, | $l$, |
| 1.5, | 2.2, | 2.6, | 3.3, | 3.5, | 3.9, | 4.0, | 4.4, | 4.6, | 5.1, | 5.3, | 5.7, |
| $m$, | $n$, | $o$, | $p$, | $q$, | $r$, | $s$, | $t$, | $u$, | $v$, | $w$, | |
| 6.2, | 6.4, | 7.1, | 7.5, | 8.0, | 8.2, | 9.3, | 11.1, | 12.0, | 13.1, | 18.0, | |

then the table shows for each deg-order, the several covariants of that deg-order, and

www.rcin.org.pl

the number of them which are asyzygetic; for instance, $i=5$ as above, $j=6$, an extract from the table is

| $j$ | $k$ | $\theta$ | | $(k+1)\theta$ |
|---|---|---|---|---|
| 6 | 30 | 1 | $a^6$ | 31 |
| | 28 | 0 | | 0 |
| | 26 | 1 | $a^4c$ | 27 |
| | 24 | 1 | $a^3f$ | 25 |
| | 22 | 2 | $a^4b$, $a^2c^2$ | 46 |
| | 20 | 2 | $a^3e$, $acf$ | 42 |
| | 18 | 3 | $a^3d$, $a^2bc$, $c^3$, $f^2$ | 57 |
| | 16 | 2 | $a^2i$, $abf$, $ace$ | 34 |
| | 14 | 4 | $a^2b^2$, $a^2h$, $acd$, $bc^2$, $ef$ | 60 |
| | 12 | 3 | $abe$, $al$, $ce$, $df$ | 39 |
| | 10 | 4 | $a^2g$, $abd$, $b^2c$, $ch$, $e^2$ | 44 |
| | 8 | 2 | $ak$, $bi$, $de$ | 18 |
| | 6 | 4 | $aj$, $b^3$, $bh$, $cg$, $d^2$ | 28 |
| | 4 | 1 | $n$ | 5 |
| | 2 | 2 | $bg$, $m$ | 6 |
| | 0 | 0 | | 0 |

$$462 = \frac{\Pi(11)}{\Pi(5)\,\Pi(6)},$$

where, for instance deg-order 6.14, the covariants are $a^2b^2$, $a^2h$, $acd$, $bc^2$, $ef$, but the number against these in the third column being (not 5 but) 4, the meaning is that there exists between these five terms one syzygy, making the number of asyzygetic covariants of the deg-order 6.14 to be 4. The second column thus in fact contains the several values of $k$, and the third column the corresponding values of $\theta$; whence, forming the several products $(k+1)$ as shown, the sum of these is as it should be $=462$.

*Cambridge*, 13 *July*, 1878.

www.rcin.org.pl