668.

ON COMPOUND COMBINATIONS.

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PROF. CLIFFORD'S paper, "On the Types of Compound Statement involving Four Classes," [volume of *Proceedings* quoted, pp. 88—101; *Mathematical Papers*, pp. 1—13], relates mathematically to a question of compound combinations; and it is worth while to consider its connexion with another question of compound combinations, the application of which is a very different one.

Starting with four symbols, A, B, C, D, we have sixteen combinations of the *five types* 1, A, AB, ABC, $ABCD$, $(1+4+6+4+1=16$ as before). But in Prof. Clifford's question 1 means $A'B'C'D'$, A means $AB'C'D'$, &c.; viz. each of the symbols means an aggregate of four assertions; and the 16 symbols are thus *all of the same type*. Considering them in this point of view, the question is as to the number of types of the binary, ternary, &c., combinations of the sixteen combinations; for, according as these are combined,

$$\text{No. of types} = \frac{1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12,\ 13,\ 14,\ 15}{1,\ 4,\ 6,\ 19,\ 27,\ 47,\ 55,\ 78,\ 55,\ 47,\ 27,\ 19,\ 6,\ 4,\ 1}$$

together.

In the first mentioned point of view the like question arises, in regard to the sets belonging to the five different types separately or in combination with each other; for instance, taking only the six symbols of the type AB, these may be taken 1, 2, 3, 4, or 5 together, and we have in these cases respectively

$$\text{No. of types} = \frac{1,\ 2,\ 3,\ 4,\ 5}{1,\ 2,\ 2,\ 2,\ 1},$$

as is very easily verified; but if the number of letters $A, B, \ldots$ be greater (say this $= 8$), or, instead of letters, writing the numbers 1, 2, 3, 4, 5, 6, 7, 8, then the question is that of the number of types of combination of the 28 duads 12, 13, ..., 78, taken 1, 2, 3, ..., 27 together, a question presenting itself in geometry in regard to the bitangents of a quartic curve (see Salmon's *Higher Plane Curves*, Ed. 2 (1873), pp. 222 *et seq.*): the numbers, so far as they have been obtained, are

$$\text{No. of types} = \frac{1,\ 2,\ 3,\ \ 4, \ldots,\ 24,\ 25,\ 26,\ 27}{1,\ 2,\ 5,\ 11, \ldots,\ 11,\ \ 5,\ \ 2,\ \ 1}.$$

It might be interesting to complete the series, and, more generally, to determine the number of the types of combination of the $\frac{1}{2}n(n-1)$ duads of n letters.