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ON THE REPRESENTATION OF A SPHERICAL OR OTHER SURFACE ON A PLANE: A SMITH'S PRIZE DISSERTATION.

[From the *Messenger of Mathematics*, vol. II. (1873), pp. 36, 37.]

IN the Smith's Prize Examination for 1871 I set as the subject for a dissertation:

The representation of a spherical or other surface on a plane.

I give the following as a specimen of the sort of answer required: an answer which, without so much as noticing that projection (in its restricted sense) is only one kind of representation, goes into the details of the constructions for the different projections of the sphere, and even into the demonstrations of these constructions, errs quite as much by excess as by defect, and is worth very little indeed.

The question is understood to refer to Chartography, viz. the kind of representation is taken to be such as that of a hemisphere or other portion of the earth's surface in a map.

An implied condition is that each point of the surface (viz. of the portion thereof comprised in the map) shall be represented by a single point on the map; and conversely, that each point on the map shall represent a single point on the surface. And further, any closed curve on the surface must be represented by a closed curve on the map, and the points within the one by the points within the other. If for shortness the term element is used to denote an infinitesimal area included within a closed curve, we may say that each element of the surface must be represented by an element of the map; and conversely, each element of the map must represent an element of the surface.

A map would be perfect if each element of the surface and the corresponding element of the map were of the same form, and were in a constant ratio as to magnitude; say if it were free from the defects of "distortion" and "inequality" (of scale); the condition as to form, or freedom from distortion, may be otherwise expressed by saying that any two contiguous elements of length on the surface and the corresponding two contiguous elements of length on the map must meet at the same angle (this at once appears by taking the two elements of area to be each of them a triangle). But for a spherical or other non-developable surface, it is not possible to construct a map free from the two defects.

An obvious and usual kind of representation is that by projection: viz. taking any fixed point and plane, the line joining any point P of the surface with the fixed point meets the fixed plane in a point P' which is taken to be the representation of the point P on the surface.

When the surface is a sphere the projection is called orthographic, gnomonic or stereographic according to the positions of the fixed point and plane: the last kind is here alone considered; viz. in the stereographic projection the fixed point is on the surface of the sphere, and the fixed plane is parallel to the tangent plane at that point, and is usually and conveniently taken to pass through the centre of the sphere.

The stereographic projection is one of those which is free from the defect of distortion; it is consequently, and that in a considerable degree, subject to the defect of inequality. It possesses in a high degree the important quality of facility of construction, viz. any great or small circle on the sphere is represented by a circle in the map; and from the general property of the equality of corresponding angles, or otherwise, there arise easy rules for the construction of such circles.

The so-called Mercator's projection is an instance of a representation which is not in the above restricted sense a projection; and which is free from the defect of distortion: viz. the (equidistant) meridians are here represented by a system of (equidistant) parallel lines; and the parallels of latitude by a set of lines at right angles thereto: the distance between consecutive parallels in the map being taken in such wise as is required to obtain freedom from distortion; for this purpose the increments of latitude and longitude must have in the map the same ratio that they have on the sphere, and since in the map the length of a degree of longitude (instead of decreasing with the latitude) remains constant, the lengths of the successive degrees of latitude in the map must increase with the latitude: the scale of the representation thus increases with the latitude, and would for the latitude $\pm 90°$ become infinite.

There is a simple representation of a hemisphere, due to M. Babinet, in which the defect of inequality is avoided, viz. the meridians are represented by ellipses having their major axes coincident with the diameter through the poles and dividing the equator into equal distances, and the parallels by straight lines parallel to the equator.