486.

NOTE ON DR GLAISHER'S PAPER ON A THEOREM IN DEFINITE INTEGRATION.

[From the *Quarterly Journal of Pure and Applied Mathematics*, vol. x. (1870), pp. 355, 356.]

It is worth noticing how easily the case when $\phi = 1$ may be proved independently of the general formula with Θ; for (1) the equation

$$v = ax - \frac{a_1}{x-\lambda_1} - \frac{a_2}{x-\lambda_2} \ldots - \frac{a_n}{x-\lambda_n}$$

is

$$(ax - v)(x-\lambda_1)(x-\lambda_2)\ldots - a_1(x-\lambda_2)\ldots - \ldots = 0,$$

and has $n+1$ roots, say $x_1, x_2 \ldots x_{n+1}$ where

$$x_1 + x_2 \ldots + x_{n+1} = \lambda_1 + \lambda_2 \ldots + \lambda_n + \frac{v}{a}$$

and (2) the equation

$$v = -\frac{a_1}{x-\lambda_1} - \frac{a_2}{x-\lambda_2} \ldots - \frac{a_n}{x-\lambda_n}$$

is

$$-v(x-\lambda_1)(x-\lambda_2)\ldots - a_1(x-\lambda_2)\ldots - \ldots = 0,$$

and has n roots $x_1, x_2 \ldots x_n$ where

$$x_1 + x_2 \ldots + x_n = \lambda_1 + \lambda_2 \ldots + \lambda_n - \frac{a_1 + a_2 \ldots + a_n}{v};$$

wherefore

$$fv\,dx_1 + fv\,dx_2 \ldots = fv\,(dx_1 + dx_2 \ldots)$$

$$= fv\,\frac{dv}{a} \quad \text{in the first case}$$

and

$$= fv\,\frac{(a_1 + a_2 \ldots + a_n)\,dv}{v^2} \quad \text{in the second}$$

[which are the two formulæ in question].