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Calculation of the supersonic flow around wings with the attached shock wave on the leading edge

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A NUMERICAL solution of the problem of the supersonic flow around wings with the attached shock wave on the leading edge is presented. The form of the wings may be triangular, rhombic-shaped or arrow-headed with the sharp tips. The wing surface may be arbitrary. The angles of attack are limited only by the detachment of the shock wave from the leading edge. Non-linear equations for nonviscous gas are used. The solution is based on the finite-difference second-order method. Some calculation results are compared with the experimental data and the data of the linear theory. Approximate functions of pressure and aerodynamic coefficients based on the data of systematic calculations are presented.

Przedstawiono rozwiązanie numeryczne dla naddźwiękowego opływu skrzydeł z falą uderzeniową występującą na krawędzi natarcia. Kształt skrzydeł może być trójkątny, rombowy bądź w formie strzały z ostrymi końcami. Powierzchnia skrzydeł może być dowolna. Kąty natarcia są ograniczone odrywaniem się fali uderzeniowej od krawędzi natarcia. Jako równania konstytutywne przyjęto nieliniowe równania dla nielepkiego gazu. Do rozwiązania zagadnienia posłużono się metodą różnicową z dokładnością drugiego rzędu. Niektóre wyniki obliczeń porównano z danymi doświadczalnymi i wynikami teorii liniowej. Dokonując systematyzacji danych liczbowych podano przybliżone zależności dla ciśnienia i współczynników aerodynamicznych.

Представлено численное решение задачи сверхзвукового обтекания крыльев с ударной волной, присоединенной к передней кромке. Форма крыльев может быть треугольной, ромбовидной или стреловидной с острыми концами. Поверхность крыльев может быть произвольной. Углы атаки ограничены только отходом ударной волны от передней кромки крыла. Используются нелинейные уравнения для невязкого газа. Решение основано на конечно-разностном методе второго порядка точности. Некоторые результаты расчетов даны в сопоставлении с экспериментальными данными и данными линейной теории. Представлены приближенные зависимости для давления и аэродинамических коэффициентов, построенные по данным систематических расчетов.

Introduction

THE PAPER deals with the problem of the supersonic flow around delta wings, swept or rhombic-shaped wings. The wing airfoil may be arbitrary. The shock wave is attached to the leading edge of the wings. The angles of attack are limited only by the detachment of the shock wave from the leading edge. The flow near the trailing edge has no subsonic regions.

The following parameters are computed: the fields of the velocity vector, pressure and density in the domain bounded by the wing surface, and the shock wave surface or outer characteristic surface.

Since the shock wave is attached to the leading edge, the flows above and under the wing do not influence each other. Therefore, the problems for upper and lower surfaces of the wings are considered separately [1, 2].

For a flat delta wing this problem has already been considered by D. A. BABAEV [3] and also by P. KUTLER and H. LOMAX [4].

1. The problem in the compression region

We take Cartesian axes (x, y, z) with origin at the apex of the wing as shown in Fig. 1. The velocity vector of the free stream is placed arbitrarily. To cut down the computer time, the velocity vector may be placed in plane of the symmetry.

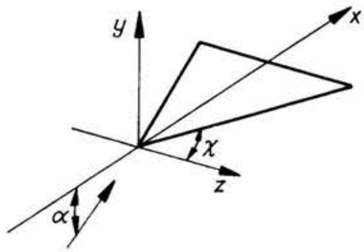


FIG. 1.

The surface of the wing is known and it is represented by the smooth function $y = G(x, z)$. The surface of the shock wave is the unknown function $y = F(x, z)$.

The flow of inviscid gas around the wing may be chemically-reactive at high temperature.

The problem is formulated for non-linear hyperbolic partial differential equations governing the flow field. The boundary conditions are set on the surface of the wing and on the shock wave. The shock wave rests on the leading edge; gasdynamical functions on the edge are assumed to be the same as on a sliding wedge.

The initial conditions are represented by gasdynamical functions and coordinates of the shock wave in plane $x = x_0$ near the apex of the wing. At first these functions are set approximately. Then, they are defined more accurately by the stationing method along coordinate x, because the surface of the wing near the apex is conical with auto-modelling flow along this coordinate. The basic algorithm is modified for this purpose.

The system of the time-independent gasdynamic equations is written in matrix form as follows:

$$A\frac{\partial X}{\partial x}+B\frac{\partial X}{\partial y}+C\frac{\partial X}{\partial z} = 0,$$

where X — vector-columns with the components u, v, w, p, ϱ; u, v, w — velocity components, p — pressure, ϱ — density. A, B, C are matrices of the fifth order. Such are the components in these matrices:

$$a_{11} = b_{12} = c_{13} = \varrho, \qquad a_{15} = a_{24} = a_{33} = a_{42} = a_{51} = u,$$
$$a_{21} = b_{22} = c_{23} = \varrho a^2, \qquad b_{15} = b_{24} = b_{33} = b_{42} = b_{51} = v,$$
$$a_{54} = b_{44} = c_{34} = 1/\varrho, \qquad c_{15} = c_{24} = c_{33} = c_{42} = c_{51} = w,$$

and the others are zero.

When considering the flow of real gas it is necessary to solve the system of equations of the thermodynamic equilibrium to compute a — the velocity of sound.

The solution of the problem is based on the numerical second-order method developed by K. I. BABENKO and the author of this article in 1961 [5].

The domain of the solution is transformed into a rectangle by introducing auxiliary variables:

$$t = x, \quad \xi = (y-G)/(F-G), \quad \theta = z/H(t), \quad 0 \leqslant \theta < 1, \quad 0 \leqslant \xi \leqslant 1,$$

where $H(t)$ is the applicata of the leading edge. This domain is divided into rectangular cells and the implicit finite-difference scheme is used. The solution is sought "step-by-step" from the plane $x = x_0$ to $x = x_0 + n\Delta x, \ldots, x = N$.

2. The problem in the expansion side of the wing

The problem is formulated and solved the same as the problem of the compression region but we have three distinctive features.

First: the boundary of the domain is not the shock wave surface, but the outer characteristic surface. Therefore the gasdynamical functions on the outer surface are equal to the functions of the free stream.

Second: the leading edge is the base of the straight characteristic in the Prandtl-Meyer flow.

Third: there is constant entropy in the domain equal to the entropy of the free stream except in the region behind the embedded shock wave. This embedded shock wave is formed because the flow after expansion on the leading edge approaches the plane of symmetry and is compressed. There is usually a small change in entropy on this inboard shock. Stability conditions of the computing in the region of that shock may be provided by the artificial viscosity terms.

3. Application of the method

The present method was employed in Ref. [6] to compute the gasdynamical functions and aerodynamical coefficients for flat delta wings, delta wings and swept wings with the biconvex airfoil and the sharp wing-tips.

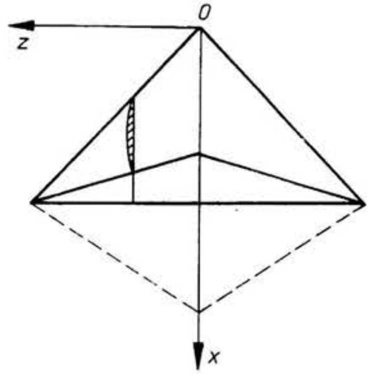


FIG. 2.

The data for the parameters $M_\infty = 2 \div 10$, angles of attack $\alpha = 0 \div 15°$ and angles of sweep $\chi = 45 \div 75°$ are given in the tables in that reference.

We can obtain data for swept-wings with sharp wing-tips from the flow around the flat delta wing with the same sweep leading edges. The flat delta wings can be transformed into flat swept-wings by the increasing the sweep of the trailing edge.

The method may be employed to compute the flow around swept and rhombic-shaped wings with the biconvex airfoil and supersonic trailing edge. For this purpose a swept wing with airfoil must be superimposed on a flat delta wing so that their apexes and leading edges coincide (Fig. 2). Then, computation is made for such a complex wing. The flow on the section behind the trailing edge of the swept wing is not taken into consideration. It does not influence the flow around the swept-wing.

4. Some results of the calculation

The pressure coefficient in the compression region on the delta wing with airfoil and its experimental data are shown in Fig. 3. The pressure upon the lower conical surface calculated by our method and by the linear theory are shown in Fig. 4. There is a large difference between linear and non-linear results at the angle of attack $\alpha = 10°$.

The isentropy lines and the shock wave in the cross-section of the conical delta wing are shown in Fig. 5.

The local pressure calculated near the upper surface of the flat delta wing in the expansion region and the pressure obtained experimentally [7] are shown in Fig. 6. The location of the embedded shock wave upon the upper surface of the flat delta wing from the calculation and the same data obtained experimentally [7] are shown in Fig. 7. The location of that embedded shock which depends on the angle of attack is shown in Fig. 8.

The upper surface of the delta wing with 5% relative thickness of the airfoil is shown in Fig. 9. It has an angle of attack of 5°. There is a compression region there but at the end of the wing near the leading edge there is an expansion region. Here, the outer shock wave is transformed into the outer characteristic surface. That transition is shown with the crosses on the shock wave.

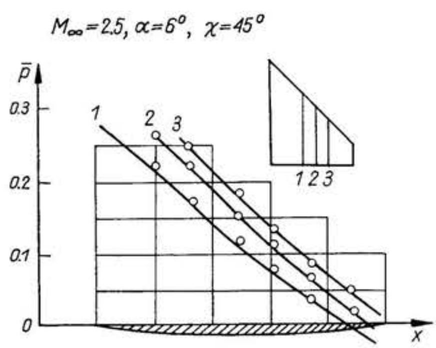
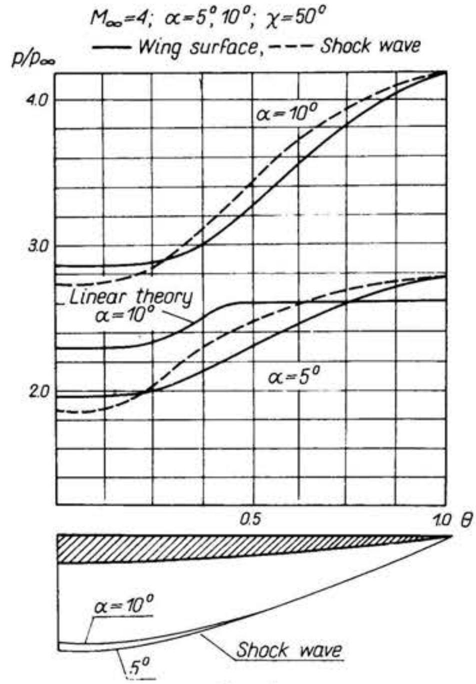


FIG. 3.



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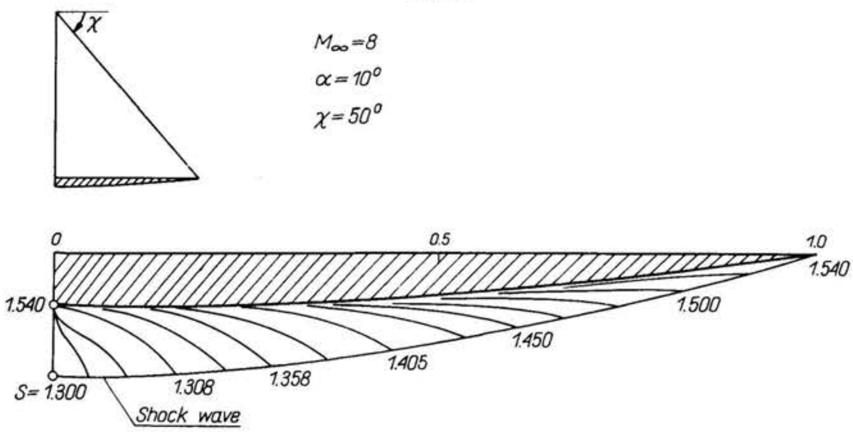

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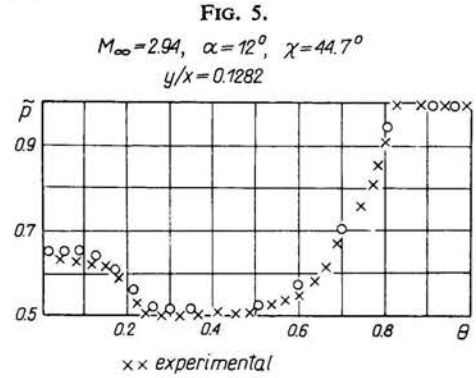

FIG. 6.

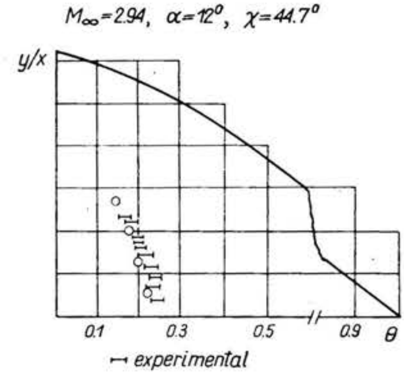

FIG. 7.

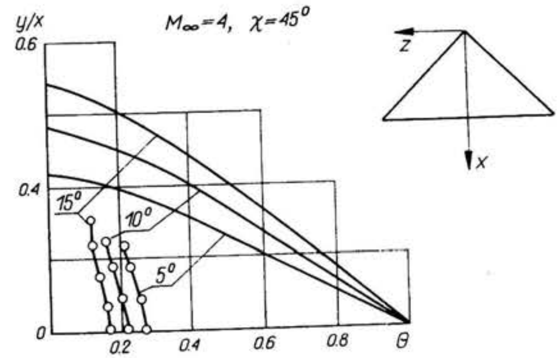

FIG. 8.

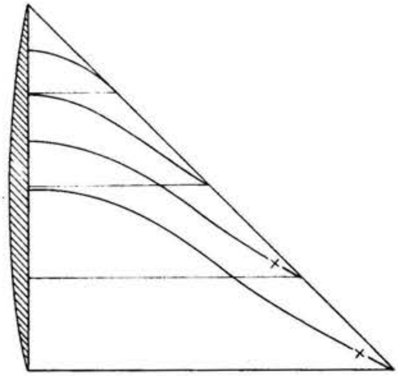

FIG. 9.

From the given calculation we obtained an approximation for the resultant force of the pressure as the correction of the linear theory

$$P_r = 1 \pm {}^1/_2(\Delta P_e)[1 \pm 0.01\, M_\infty (57.3\alpha)^{1\pm 0.015 M_\infty}].$$

For the lower surface "+" and for the upper surface "−", where $\Delta P_e = 0.7 M_\infty^2 \dfrac{4\alpha}{\sqrt{M_\infty^2 - 1}}$ by the linear theory.

From this we can obtain the coefficient of the normal force

$$c_n = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}} (1 + 0.05 M_\infty^2 \alpha^{1.5}).$$

Acknowledgment

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