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AN ANALYSIS OF PATTERNS OF VARIATION IN SOME REPRESENTATIVE *MAMMALIA*. PART III. SOME EQUATIONS ON THE NATURE OF FREQUENCY DISTRIBUTIONS OF ESTIMATED VARIABILITIES

ANALIZA WZORCOW ZMIENNOŚCI NIEKTÓRYCH PRZEDSTAWICIELI *MAMMALIA*. III. RÓWNANIA CHARAKTERYZUJĄCE ROZKŁADY ZMIENNOŚCI

Frequency distributions of mammalian coefficients of variation are mathematically defined as a rational function $y = R(x)$, where $R(x) = P(x)/Q(x)$, and x is always positive. The equation is differentible, and the modal x is twice the theoretical value of $X_1(=k)$. An inflection point exists at $3k$, which is a practical point of defining »eccentric« variations some of which are mentioned. The height of y depends in part on the sample size, which view has been overlooked.

In nature, as well as in society, eccentric phenomena exist which upon investigation prove important and interesting. Estimates of variability (coefficients of variation) in selected measurements of a representative sample of the *Mammalia* (Long, 1968, 1969) formed frequency curves skewed to the right. The curve obviously reflected graphically some axiomatic generalizations including such statements as Darwin's (1859) famous postulate that all organisms vary, and the observation of Simpson (1953) and others that coefficients of variation of morphological structures usually range from 2—6 or 2—8, and that larger coefficients are of »vestigial« structures of those which do not require definite form. The tails and slope of the frequency curves suggested information about mammals, at least, on the theoretical nature of variability.

In this paper an attempt is made to determine some general equations that reasonably fit the observed skewed curves. Such an approach provides mathematical insights and distinguishes eccentric populations from those that »properly« range in variation near the mode value. The differentiable equations permit determination of the lowest or modal estimates of variability. Acknowledgments are in Part I (Long, 1968).

The curves seem based on a rational function $y = R(x)$, where $R(x) = P(x)/Q(x)$. The limit of $|y|$ as $|x| \to \infty$ is zero, and $y = 0$ is an horizontal asymptote. In the frequency distributions $x > 0$ when $y = 0$. Therefore,

$$\lim_{|x| \to \infty} |y| = \frac{(m)\,(|x| - k)}{|x|^2} = 0 \qquad (1)$$

When $|y| \to 0$ then $|x| \to \infty$ or $|x| \to k$, and $|y|$ cannot $\to \infty$ because $|x|^2 > 0$. Large values of x decrease the dependent variable y, and were it not for the constant m, the maximal height of y would never exceed one. The value k is based on Darwin's postulate that all populations vary. In the skewed curves x_1 is always a small positive number, and if there is a point of inflection to the left of the mode it seems unimportant.

For $y = f(x)$ defined for the range $k \leqslant x < 0$,

$$\frac{d}{dx} \frac{(m)(x-k)}{x^2} = \frac{2mkx - mx^2}{x^4} \tag{2}$$

$$= \frac{m(2k - x)}{x^3} \tag{3}$$

And if $x - 2k = 0$, then $x = 2k$ at the highest point on the curve. The modal frequency for coefficients of variation is at $2k$, and k is x_1 which is the x for the frequency of the theoretical minimal coefficient. If the modal frequency and k are known, m can be calculated in equation (1), when the modal frequency is substituted for y and the x of the mode is $2k$.

Furthermore,

$$\frac{d^2}{dx^2} \frac{m(2k - x)}{x^3} = \frac{-x^3(m) - 6x^2k(m) + 3x^3(m)}{x^6} \tag{4}$$

$$= \frac{2m(x - 3k)}{x^4} \tag{5}$$

When $x - 3k = 0$, then $x = 3k$. Thus, a point of inflection of the curve occurs to the right of the mode ($2k$) at $3k$. Coefficients of variation greater than $3k$ are defined as eccentric, and populations that show such high variation should be investigated to determine its cause.

Inasmuch as $y = f(x)$, use of equation (1) reveals that when x is $3k$ the corresponding value of y is $2m/9k$ which is the value of y_3.

The instantaneous slope $\Delta y/\Delta x$ at the point of inflection (y_3, $3k$) is determined by use of equation (3) to be $dy/dx = -m/27k^2$. Can it be true that the magnitude of k is directly related to the magnitude of general variability? The frequency distributions (Long, 1969) support this hypothesis, evidenced by the frequency curves for low and high values of k (*e.g.*, length of skull and total length).

The nature of frequency distributions of variations in other kinds of organisms should be appraised. In birds most estimates so far measured (Bader & Hall, 1960) indicate low variability, and perhaps owing to the problems of flight in birds (and bats, Long, 1969; Bader & Hall, 1960; Long & Kamensky, 1967) high coefficients are seldom if ever seen. Therefore, the equations above perhaps apply to bird populations.

The height of y is dependent upon m or k in respect to the interval x, including the peak of y at $x = 2k$. Therefore, the reported significance of the leptokurtic curves (Long, 1969) now must be ascribed partly to the effect of m. This value depends upon the sample size, for the higher frequencies obviously increase faster than the lower ones. This explains why the curves based on smaller samples (of combined taxa) are lower in Long's figures than are the curves for the larger samples. The abundant values available for length of skull produced a markedly leptokurtic curve, and its sharp peak resulted in part from the numerous values. The reason for the heights of the sharp peaks was, unfortunately, overlooked by Long until they were investigated algebraically.

SOME ECCENTRIC MAMMALS

The compilation of wild placental coefficients of variation (Long, 1968) was used to determine some modal, then eccentric values. Data listed by Yablokov (1966) do not lend themselves readily to this purpose, although it was possible in some cases to estimate modes, and to list genera that have some eccentric values. He found that *CV's* varied remarkably in many genera, and therefore mean *CV's* or at least typical ones, when available, are desirable. Long (1969) found that high variation in mammals is related to domestication, large size, and a marine niche, which factors are obviously important in this analysis of the same data and of Yablokov's samples. Domestic mammals are not listed below.

Concerning total length, eccentric values exceed six. Some populations vary chiefly in length of tail (*e.g.*, *Peromyscus truei*) when they vary in total length. *Phyllotis, Microtus pennsylvanicus, Rattus rattus*, and *Physeter* seem eccentric. Body lengths listed by Yablokov seemed to have a mode of four, and some eccentric samples were found in the following genera: *Sorex, Erinaceus, Neomys, Talpa, Apodemus, Sicista, Clethrionomys, Spermophilus* (= *Citellus*), *Pusa, Pagophilus, Cystophora, Hystriophoca, Phoca, Callorhinus*, and *Vulpes*.

Concerning length of skull, eccentric values exceed three, and the samples include *Ateles* (males), *Ochotona, Thomomys talpoides* (males, only 13 specimens), some *Phyllotis, M. pennsylvanicus, Ondatra, R. rattus, Physeter, Balaenoptera, Ursus arctos, Canis lupus, Mephitis mephitis, Lutra lutra, Odobenus, Odocoileus*, and some *Rangifer*. The high variation in *Ochotona* is inexplicable. The mode for Yablokov's values is between two and three, apparently closer to the latter. Thus eccentric values might begin at 4.5 instead of 3. Eccentric samples are seen in *Talpa, Erinaceus, Apodemus, Lagurus, Clethrionomys, Lynx, Ursus, Lutra, Pusa*, and *Cystophora*.

Concerning cranial breadth, eccentric values exceed 4.5, and include those of marine *Balaenoptera, Eumetopias* (small sample) and *Odobenus*. Yablokov's mode might be as low as two, and eccentric values (exceeding three) are found in some *Apodemus, Castor, Erinaceus, Aluoatta, Ateles, Tursiops, Mustela, Ursus, Lutra, Pusa, Callorhinus, Cystophora*, and *Pagophilus*. Of these, *Erinaceus, Alouatta*, and *Mustela* are not eccentric when $3k$ is 4.5.

Concerning maxillary tooth-row, eccentric values exceed six, and are found in male *Thomomys*, some populations of *Dipodomys* (the observed range is so small the *CV*'s are suspect), one small sample of *Canis lupus*, and small samples of *Eumetopias* and *Hydropotes*. No conclusions seem warranted.

Concerning interorbital length, eccentric values exceed 4.5, and are seen in *Ochotona, Thomomys, M. pennsylvanicus, Ondatra, R. rattus, Zapus hudsonius* (another surprise), *Ursus arctos, C. lupus, L. lutra, Lynx rufus, Hydrurga* females, and both sexes (small samples) of *Eumetopias*.

Concerning zygomatic breadth, eccentric values exceed six and are seen in *Tupaia, M. pennsylvanicus*, some populations of *C. lupus, L. lutra, Hydrurga* females, and *Odobenus.*

To sum up, by making frequency distributions for each kind of measurement and using the inflection point at $3k$ to define eccentric variations, resolution for categorization of high estimates of variability is possible. Furthermore, $3k$ is not merely an arbitrary and practical value, but is derived from a general equation which seems to describe the empirical frequency distributions, and is the theoretical point at which the frequency curve changes from convex to concave. Other considerations of the general equation reveal that the height of the peak depends in part on the sample size, and that the mode and variability in general appear related to the theoretical minimal estimate of variability.

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MARKING BEAVERS

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Experiments with marking 10 beavers by depigmentation of frost-bitten sites were made in 1969. Marking was carried out with cast iron numbering stamps cooled in liquid nitrogen (—195.8°C) and applied to the beaver's tail. Depigmentation in the form of legible numbers was obtained within a period of 3—9 weeks from the time of marking. The way in which marking was carried out and the animal s age affected the rate of appearance of depigmentation.

Lasting and legible marking of beavers is an extremely important operation in scientific work and breeding practice, although the methods hitherto used are not fully satisfactory. Web punching between the digits of the hind feet (Aldous, 1940), tail branding and fixing rivets on the